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THE GROWTH OF SEEDLINGS OF THE CANTELOUP, CUCUMIS MELO, IN THE ABSENCE OF EXOGENOUS FOOD AND LIGHT

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In the normal course of growth of a higher plant, the energy and material used in the early stages of development are derived from the cotyledons of the seed. Soon, however, under the influence of light the seedling develops chlorophyll, which synthetically utilizes the CO_2 of the air as food while the roots absorb water and mineral substances. The energetics of such a living organism involve an irreversible progression in which three basic elements are distinguishable, as shown in the following *schema* (Pearl[1]).

A →	*B* →	*C*
External Sources of Energy and Non-Living Matter (Food. Either chemical compounds of external origin which yield energy and utilizable material when split by the organism (animals), or similar compounds synthesized by the organism itself, with energy derived from light and subsequently split to yield energy and utilizable material.)	*The Organism* (An aggregation of matter which has the property of autonomously changing *A* to *C*, *by virtue of its pattern or organization*.)	*The Product* (Either (*a*) heat, (*b*) work, (*c*) component matter of the organism or (*d*) waste, meaning chemical compounds incapable of yielding energy with economic profit by further splitting.)

The cotyledons of the seed of a plant like the canteloup (*Cucumis melo*), and the stored nutriment which they contain are an integral part, and an important part of the total organic pattern of the individual. If now we sterilize such a seed, plant it on a medium which contains no food material upon which the roots can draw, and keep the whole preparation in the dark, the growth of the etiolated seedling which ensues is an expression of the

inherent vitality of that individual, since it must draw whatever nutriment it gets from endogenous sources, which are themselves an integral part of the total organic pattern of the individual. *Inherent vitality* is then defined as the total potential capacity of an organism to perform vital actions in the complete absence of exogenous derivation of food or energy.

During the past two years we have carried out numerous experiments with canteloup seedlings grown under sterile conditions in the absence of exogenous food and of light, so that the growth depended only on matter present as a part of the inherent organization of the plant plus sterile distilled water obtained through the roots. The testae of the seeds were removed (see Pearl and Allen[2]) and the shelled seeds, after sterilization by immersion for one minute in 1:1000 $HgCl_2$ solution, were soaked in sterile distilled water for three hours. Each seed was then planted on sterile agar jelly ($^1/_2$ per cent agar in sterile distilled water) in long glass tubes which were then tightly corked and put in light-proof boxes in an incubator at 30°C. Each day the length of each sprout was measured by means of a millimeter scale placed against the outside of the tube. This was done in a dark room, lighted only by a photographically non-active ruby lamp. Complete details of the procedure are given by Pearl.[3]

Let us examine the results of a series of experiments in which 50 tubes were involved. Each day during the experiment the air in each tube was changed under aseptic precautions. The experiment originally started with 54 tubes, of which 50 came through the entire run without contamination. The mean lengths of these 50 stems at each observation are shown in table 1. Days of growth are reckoned from the planting of the seeds, May 13, 1927, 4 P.M.

TABLE 1

MEAN LENGTH OF THE STEMS OF CANTELOUP SEEDLINGS GROWN IN THE ABSENCE OF EXOGENOUS FOOD AND OF LIGHT

DAYS OF GROWTH	OBSERVED LENGTH CM.	CALCULATED LENGTH CM.	DAYS OF GROWTH	OBSERVED LENGTH CM.	CALCULATED LENGTH CM.
2.92	3.78	4.0	9.85	20.57	20.7
3.88	6.92	6.8	10.85	20.93	21.1
4.92	10.98	10.5	11.79	21.26	21.2
5.88	14.09	14.1	12.92	21.33	21.3
6.83	16.59	16.9	13.88	21.36	21.4
7.79	18.50	18.8	14.79	21.38	21.4
8.83	19.80	20.1	16.88	21.38	21.4

The simple logistic curve,

$$y = \frac{21.38}{1 + e^{3.487 - 0.704x}},$$

was fitted to the observed mean stem lengths. The columns of table 1 headed "Calculated length" are the ordinates (y) of the theoretical curve, at time x.

The observed points (circles) and the calculated (smooth curve) are shown graphically in figure 1. It is obvious that the logistic gives a close graduation of the data.

The results of another experimental series, conducted in the same manner as that described above except that the tubes were not aerated daily, are

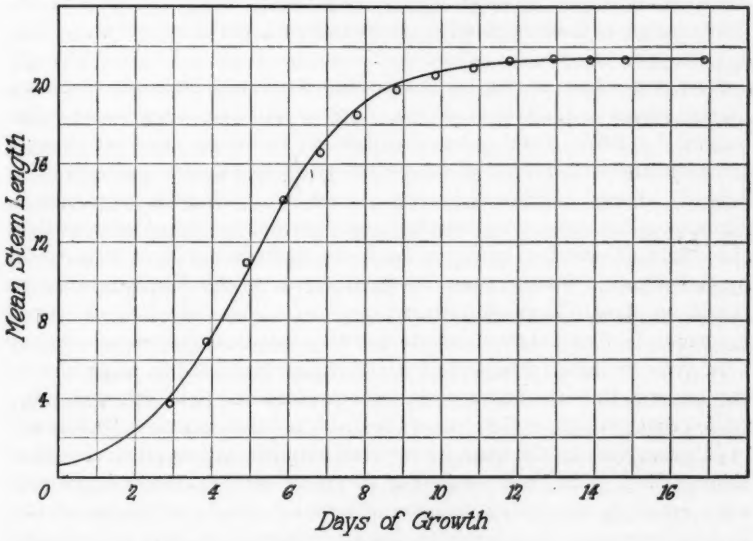

FIGURE 1

Observed and calculated mean stem length of seedlings of *Cucumis melo* grown in the complete absence of exogenous food and light.

shown in table 2. Days of growth are reckoned from the planting of the seeds, January 18, 1927, 4.30 P.M.

TABLE 2

MEAN LENGTH OF STEM OF CANTELOUP SEEDLINGS GROWN IN THE ABSENCE OF EXOGENOUS FOOD AND OF LIGHT

DAYS OF GROWTH	OBSERVED LENGTH CM.	CALCULATED LENGTH CM.	DAYS OF GROWTH	OBSERVED LENGTH CM.	CALCULATED LENGTH CM.
0.97	0.44	0.3	10.74	23.39	23.4
1.91	0.73	1.2	12.74	23.64	23.7
2.70	3.26	2.9	13.78	23.68	23.7
3.71	8.18	6.7	14.78	23.72	23.7
5.80	16.51	16.1	15.82	23.73	23.7
6.77	18.79	19.0	16.84	23.73	23.7
7.77	20.80	21.0	19.96	23.74	23.7
8.78	22.14	22.2	20.90	23.74	23.7
9.93	23.10	23.1	21.94	23.74	23.7

The skew logistic curve,

$$y = \frac{23.74}{1 + e^{5.8705 - 1.8193x + 0.1585x^2 - 0.0071x^3}},$$

was fitted to the observed means. Except for the observation at 3.71 days the curve fits the mean lengths very satisfactorily. There was evidently some undiscovered error connected with the observations on that day.

Now it has been shown by Reed[4,5] and Reed and Holland[6] that such diverse plants as *Helianthus*, *Pyrus*, *Juglans* and *Zea*, when grown under natural conditions, with normal derivation of exogenous food through the roots and with normal photosynthesis, follow a logistic curve in their growth. These conditions imply *total vitality* ($A + B$ of the diagrammatic schema). The same thing has also been shown to be true of the growth of the fruits of another nearly related cucurbitaceous plant (*Cucurbita*) by Robertson.[7] Furthermore, we have shown in this Institute[8] that the canteloup plant (*Cucumis*) itself, when grown under normal conditions of exogenous food and photosynthesis, follows a logistic curve in its growth.

In view of these facts, and of the evidence presented in this paper regarding growth in the absence of exogenous food and light, it may be concluded that not only is the form of the distribution of individual differences in inherent vitality the same as for total vitality, as has previously been shown (Pearl[1,3]), but also *the form of the growth curve is essentially the same when growth is the expression either of inherent vitality or of total vitality.* In both cases (so far as concerns *Cucumis melo*) the growth follows some form of the generalized logistic curve

$$y = \frac{K}{1 + e^{a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots + a_nx^n}}.$$

[1] Pearl, R., "On the Distribution of Differences in Vitality among Individuals," *Amer. Nat.*, **61**, pp. 113–131, 1927.

[2] Pearl, R., and Allen, Agnes L., "Notes on the Growing of Seedlings for Physiological Experimentation," *Proc. Soc. Expt. Biol. and Med.*, **24**, pp. 439–444, 1928.

[3] Pearl, R., *The Rate of Living*, Alfred A. Knopf, Inc., New York, **1928.**

[4] Reed, H. S., "Growth and Variation in *Helianthus*," *Amer. Jour. Bot.*, **6**, pp. 252–271, 1919.

[5] *Id.*, "The Nature of the Growth Rate," *Jour. Gen. Physiol.*, **2**, pp. 545–561, 1920.

[6] Reed, H. S., and Holland, R. H., "The Growth Rate of an Annual Plant *Helianthus*," *Proc. Nat. Acad. Sci.*, **5**, pp. 135–144, 1919.

[7] Robertson, T. B., "The Chemical Basis of Growth and Senescence," Lippincott, Philadelphia, **1923**, pp. viii + 389.

[8] Unpublished observations.

SOME FACTORS IN ROCK METAMORPHISM

BY DAVID WHITE

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Read before the Academy October 20, 1927

A review of the regional carbonization of coals and other carbonaceous deposits shows that the progressive transformation of peaty deposits and other sediments of organic composition to bituminous coals, anthracites, and finally to graphites, or to the corresponding ranks of carbonaceous matter in other types of rock, is true rock metamorphism and is attended by changes in the accompanying inorganic sedimentary deposits. The alteration of the carbonaceous sediments, which is denoted by the progressive elimination of "volatile matter" offers, in fact, a scale for the determination of the successive stages of the incipient metamorphism, the infra-metamorphism, of sedimentary rocks in which metamorphic criteria such as secondary minerals and recrystallization, usually employed by petrologists, are not generally observed until the accompanying coals have reached an advanced stage of carbonization.

Continued studies of the structural, stratigraphic and physical conditions attending carbonization prove that it is the product chiefly of horizontal thrust, temperature and time. Further examination of the effects of competency (the ability of the strata to withstand the applied thrust without buckling) clearly shows that in horizontal formations the greatest regional carbonization is found where competent strata have endured greatest thrust pressure without folding or overthrust, though equal or greater carbonization may be found in a belt of folded and faulted beds situated between the competent region and the source of the thrust. In the latter case competency has failed under superior force, but after the more advanced alteration was effected. In some instances it would appear that the thrust was mainly carried beneath superficial upward dislocated blocks. Buckling and over-thrusting of the strata absorb the thrust, relieving the pressure and averting its greatest intensity as well as its full duration. Accordingly, the coals involved in the steep folds and for some distance behind the thrust faults are less altered than those in more competent rocks along strike subjected to approximately the same force, the differences being found to correspond, in general, to the extent to which the arc is shortened and the thrust compensated by the folding and thrust faulting, and, apparently, to the depth of the displacements. Isometamorphic lines or "isocarbs," may run at a high angle to the strike or even partly around the fold or fault, and may cross broad arches in trends strikingly incompatible with a theory of batholithic heat as the cause of the carbonization.

To an extent, thrust pressure, temperature and time may one be substituted for another. The substitutions between pressure and temperature are important, particularly as intensities increase; and time has some value as a substitute for both. However, neither the relative values of these factors, and their varying substitutional values, nor the extent to which substitutions may be made have yet been determined. The problem is complicated by variance both in conditions of alteration and in composition of the organic deposit. Time, though geologic in length, appears by itself to be of distinctly minor value as a factor but becomes important in the presence of advanced pressures and their consequent temperatures. Temperature, including that "normal" to depth of burial of a particular deposit, plus temperature of granular and molecular readjustment and chemical change consequent to the thrust, is not likely to have exceeded 300°C. in the average mined bituminous coal, most of which in all probability has never been buried 7000 feet deep. Increased thrust pressures are believed by the author to have caused cementation in anthracites at temperatures very much lower than 450°C., the approximate degree at which most bituminous coals fuse in the laboratory. Temperatures in regions geologically long quiescent are low. The drill hole near Loveland, Colorado, showed but 140°F. at a depth of 6500 feet. We do not anywhere know actual "normal" temperatures.

The conclusion is reached that the factor of preëminent importance in the metamorphism of coal and other sediments on a regional scale, independent of igneous influences which, in general, are merely local, is horizontal thrust. This thrust is exerted variably with intermittently increasing intensity through long geologic periods, not only especially through diastrophic revolutions but also during periods of relative quiescence, under the great power derived from the movement of a mass of the crust of the earth above the zone of flow against another mass.

In comparison with the magnitude of the horizontal pressures generated under thrust as regional metamorphism progresses the vertical pressure on a given coal bed due to the thickness of overlying rock is relatively insignificant. The rôles of the column of superimposed strata are (1) through its strength and weight, to promote competency and (2) through its thickness to raise the temperature in the coal according to the geothermal gradient, which steepens in regions of active horizontal compression.

The author concludes that the observed general increase in carbonization of the organic sediments in passing downward in any normal vertical section, according to the "law of Hilt," is very little influenced by mere weight of loading beyond the earliest stages of infra-metamorphism, but that thickness of loading plays a minor rôle through the geothermal gradient, which, however, also seems to lose relative effect as the alteration

under horizontal thrusts becomes more and more advanced. This is consistent with the observed diminution of the differences in the carbonization of coals lying at different depths in the vertical column as the regional carbonization passes through the semibituminous and anthracite ranks.

Since neither the relative values of the factors, pressure, temperature and time, or their respective substitutional values in the different cases, are known, and since the organic deposit itself consists of mixtures of compounds, many of which are undetermined as also are many of the chemical changes ensuing, any attempt at an empirical formula of metamorphism is purely speculative. "Normal" regional temperature probably is of little value, though temperatures generated under thrust gain importance as the thrusting becomes more intense. The product of thrust into competency becomes of paramount importance, both directly and indirectly, as the alteration progresses.

LATE TERTIARY THRUST FAULTS IN THE MOJAVE DESERT, CALIFORNIA

BY D. F. HEWETT

U. S. GEOLOGICAL SURVEY

Read before the Academy April 26, 1927

The purpose of this statement is to set forth briefly recent observations on an unusual thrust fault in the Mojave Desert and to interpret its probable significance in the geologic history of the region. Since 1921 the writer has devoted sixteen months to actual field work in the eastern Mojave Desert, but the fault referred to was only found in November, 1926. Plans contemplate the early completion of geologic mapping of the Ivanpah quadrangle in which the fault has been studied, but several years may elapse before the report on that area is published.

Thrust Faults of Southern Nevada.—Thrust faults were first recognized in southern Nevada by R. B. Rowe and J. E. Spurr[1] in 1899, but at that time their dip and significance were not determined. Little geologic work was done subsequently in the region until 1919, when in mapping the Muddy Mountains northeast of Las Vegas, Longwell[2] discovered extensive flat thrust faults. During 1921 and 1922, in studying the Goodsprings quadrangle southwest of Las Vegas, the writer mapped three extensive thrust faults in the Spring Mountain Range.[3] Since 1922 Longwell has been engaged in geologic mapping of the Las Vegas

quadrangle and the writer, the Ivanpah quadrangle, each about 3800 square miles.[4]

The outstanding conclusion concerning the structural geology derived from this recent work is that the Spring Mountains together with the adjacent and nearby ranges contain at least six extensive overthrust faults and numerous minor thrusts as well as normal faults. The traces of these faults on the surface indicate that they extend north and northeast for great distances, 50 to 100 miles or more and that they dip largely westward at low angles which range from 5° to 45°. Along these profound fractures, rocks that range from pre-Cambrian granite gneiss to Pennsylvanian limestones are thrust generally eastward upon younger rocks that range from lower Paleozoic to Jurassic. In some cases the overlying blocks have moved great distances over the underlying blocks, at least 10 miles and possibly much more. A review of all of the features and regional relations of these thrust faults indicates that they were formed in early Eocene time, but as Cretaceous and Eocene stratified rocks appear to be absent from this region, some uncertainty concerning the time of the thrust faulting must remain for a while. The exposures in several areas, notably the Muddy Mountains and Mesquite Valley, show clearly that these thrust faults were part of a group of deformations accomplished before the deposition of volcanic ash, sands and gravels, probably of Miocene age.

Shadow Mountain Thrust Fault.—The structural features to which particular reference is made here, however, are younger than the Miocene volcanic ash and gravel, and mark a second period of Tertiary thrust faulting.

The Ivanpah quadrangle is an area 56 by 68 miles, or about 3800 square miles, of which 1200 are in Nevada and 2600 in southeastern California. It extends from Piute Valley on the east nearly to Soda Lake, one of the sinks of the Mojave River, on the west and from Potosi Peak on the north to Providence Mountain on the south. The critical observations were made in three areas that form a broad northeast-trending belt in the northwest quarter of the quadrangle.

The northeastern area includes an isolated ridge two miles long and 700 feet high at the north end of Mesquite Valley, which lies between Spring Mountain on the east and the northern extension of Clarke Mountain on the west. In both of these mountains there are thrust faults of the early group, but the valley contains a sheet of mid-Tertiary sediments which locally on the west side of the valley overlap the highly folded lower Paleozoic dolomites. Although the rock exposures on the ridge are convincing, explorations for borax, which is elsewhere characteristic of the Miocene beds, fortunately confirm the conclusions stated below. The ridge is made up of two parts, the base which is thin-bedded white

volcanic ash trending generally N. 80°W. and dipping 60° to 80° north, and the apex which is a block of lower and upper Mississippian limestone about 3000 feet long, 600 feet wide and 500 feet high. These beds trend slightly north of west, dip 70° north and about 600 feet of beds are present. The surface of contact of the Paleozoic limestones and underlying ash is clearly exposed for hundreds of feet on the south and north sides and on the east end. These exposures indicate that the upper block has the form of a huge ship which has been pushed out upon the beds of ash, probably from the west.

The second area is a group of low hills, scarcely 3 miles in diameter, which lies 14 miles due south of the isolated ridge described above, and 10 miles west of Mesquite Dry Lake. This dry lake lies at an elevation of 2550 feet at the south end of Mesquite Valley, whereas the hills rise above a nearly flat surface at an elevation of about 4000 feet. There are five hills in the area, three of which range from 300 to 1200 feet in diameter, but another is a mile long and the last is two miles long. Each of these hills is made up of blocks of lower Paleozoic (largely Cambrian)

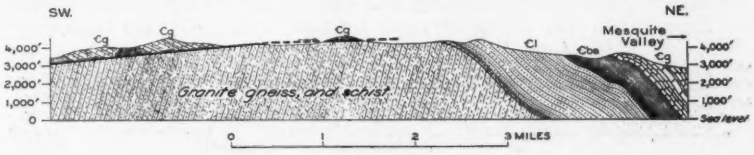

FIGURE 1

Geologic section N. 42° E. across the southwest border of Mesquite Valley. €l = Lower Cambrian quartzite, €ba = Bright Angle Shale, €g = Goodsprings formation, largely upper Cambrian but possibly in part Ordovician and Silurian.

dolomite and shale that rest upon a flat surface cut across pre-Cambrian granite gneiss. Since these beds are normally underlain by about 1200 feet of Cambrian shale, which in turn rests on 3600 feet of early Cambrian dolomite and quartzite, and the beds dip steeply into the base of granite gneiss, it is clear that the blocks have been thrust from a distant westward source to their present position. The normal succession of Cambrian beds in this region is well shown resting on the granite gneiss several miles northeast of the hills. Figure 1 shows a cross-section through these hills.

The third area is about 12 miles in diameter and lies southwest of that just described. Shadow Mountains, the largest range in it, are separable into two parts, the southern part being a single ridge four miles long and a mile wide, and the northern part, a group of isolated hills four miles long. These mountains lie 12 miles southwest of the longest hill in the second area and, like it, rise 400 to 700 feet above the nearby valleys which range in elevation from 3500 to 3200 feet. In addition to Shadow

Mountains, the area contains numerous isolated hills 300 to 600 feet high. This third area is distinctly more rugged than the second and the exposures clearly display a more convincing record.

The southern part of Shadow Mountains is a block of pre-Cambrian granite gneiss locally capped by basal Cambrian quartzite and dolomite. For a mile along the southwest slope, it is clearly shown that the granite gneiss rests on pale yellowish bentonite, volcanic ash and gravels. The base of the granite gneiss is a nearly plane surface that trends southeast and dips gently northeast, thus cutting slightly across the bedding of the underlying ash which trends nearer south and dips more steeply east. The base of the block of granite gneiss is not only smooth and flat but is not brecciated. The northern part of Shadow Mountains shows numerous blocks, rarely more than 2500 feet long and 350 feet thick, made up of Cambrian sediments that rest on ash and gravels. In a large area west of Shadow Mountains, there are numerous hills 200 to 500 feet high which show caps of either pre-Cambrian granite gneiss or Cambrian sediments resting on ash or gravels.

It seems quite clear in this area that these isolated blocks of gneiss and

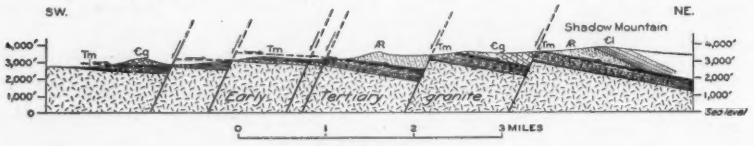

FIGURE 2

Geologic section N. 62° E. through Shadow Mountain and the hills west of it. $\mathfrak{C}1$ = Lower Cambrian quartzite, $\mathfrak{C}g$ = Goodsprings formation, largely upper Cambrian but possibly in part Ordovician and Silurian. Tm = Miocene (?) gravels, shale and volcanic ash, AR = Archean granite-gneiss and schist.

Paleozoic sediments are remnants of a once continuous sheet of rocks that were thrust over mid-Tertiary ash and gravels, but that erosion has removed a large part of the block. It is surprising that the Tertiary beds are so slightly disturbed. The local relations of numerous groups of blocks is such that one must conclude that before the thrust block was greatly eroded, it was broken by parallel normal faults that trend northwest along which the successive blocks southwest have dropped several hundred feet. The structural relations in this area are shown in figure 2.

Regional Relations.—The three areas described above form a broad arc about 30 miles long, concave toward the northwest. Obviously, the three areas present structural similarities and are related. They are parts of a great composite block of rocks that have been thrust generally eastward upon a fairly smooth surface cut across rocks as young as Miocene. The lack of distortion in the underlying beds suggests that the block moved across a surface of erosion.

From the known extent of the thrust considered with the great range in age of the rocks involved, one must conclude that it extends over a much larger area and that it may be one of the major fractures of the western states. Too little is known of the geology of the region north of the area where the thrust is exposed to hazard a guess concerning its approximate location. On the other hand, it may be related to features that have been traced throughout a large area to the west. Geologists who have worked in the northern part of the Mojave Desert since 1909 (Hess, 1909, Baker, 1911, Gregory and Noble, 1922, Hulin, 1922) have noted an impressive fault that has been traced from the western end of the desert, 150 miles northeast, to a point north of the Avawatz Mountains, or about 50 miles west of the Shadow Mountains. It has been called the Garlock fault by Hess[5] and appears on the fault map of California recently issued by the Seismological Society of America. It has commonly been considered a normal fault, but on the basis of certain displaced rock contacts, Hulin[6] inferred a much larger horizontal than dip movement. He concluded that the south side had moved eastward three miles, even though many northward-trending ranges north of the fault turn abruptly westward near it so as to suggest that the north side had moved eastward.

If the Garlock fault extends eastward on its strike along the north side of the Avawatz Mountains, it will meet the area west of the Shadow Mountains. Although conscious of the uncertainties that are attached to such correlations in a region about which little is known, the writer suggests that the faults are continuous and that instead of being a normal fault as assumed heretofore, the Garlock fault is a flaw along which the north side has moved eastward as well as upward. The eastward drift may amount to 20 or 25 miles but assuredly is more than 10 miles. If this be true, the Garlock fault may be vertical near Randsburg but would have progressive lower dips eastward. According to this explanation, the Shadow Mountain thrust marks the southeast corner of a large block that has moved eastward. To carry speculation farther, such major north-south valleys of the southern part of the Great Basin, as Death Valley, some of which are known to be limited by faults, have largely formed since upper Miocene or during Pliocene or Quaternary time. It may be recalled that Louderback[7] on the basis of work in western Nevada, concluded that the evidence points "to a late Pliocene or post-Pliocene time for the beginning of the faulting."

[1] Spurr, J. E., "Descriptive Geology of Nevada South of the Fortieth Parallel," *U. S. Geol. Survey Bull.*, **208**, pp. 175, 176, 177, 1903.

[2] Longwell, C. R., "Geology of the Muddy Mountains, Nevada," *Amer. Jour. Sci.*, **50**, pp. 39–62, 1921.

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[7] Louderback, G. D., "Period of Scarp Production in the Great Basin," *Univ. Cal. Pub. Dept. Geol. Sciences, Bull.*, **15**, p. 38, 1924.

THE QUANTUM LEVELS AND RESULTING CONSTANTS OF THE HYDROGEN MOLECULE

By Raymond T. Birge

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Communicated November 16, 1927

Richardson[1] seems to have obtained the much-sought-for connection between the visible and ultra-violet bands of hydrogen. The resulting value of the ionization potential, as recalculated by the writer from Richardson's data, is 15.34 volts. Richardson seems to think that such a low value cannot be reconciled with the observed value of about 16 volts. A similar discrepancy occurs several times in Richardson's earlier work.

It appears to the writer quite certain that the discrepancy is only apparent. Burrau[2] has calculated the total energy of H_2^+, on the basis of the new wave mechanics, as 1.204 R, or 16.30 ($\pm$ 0.03) volts, since $R = I_A =$ 13.54 volts, the ionization potential of the hydrogen atom.[3] This total energy of H_2^+ is the ionization potential of the ion, which we shall denote by I'_M. Assuming Witmer's spectroscopic value for the heat of dissociation of the neutral molecule, D = 4.34 volts, we have for the ionization potential of the neutral hydrogen molecule, $I_M = D + 2I_A - I'_M$ = 15.12 volts, as compared to the above value of 15.34 volts. Furthermore, Condon,[4] using Burrau's result and making certain reasonable assumptions, has calculated the total energy of H_2 as approximately 2.325 R, or 31.48 volts. This gives D = 4.40 volts, and I_M = 15.18 volts (31.48 − 16.30). In addition, Condon shows that the most probable transition from the normal, non-vibrating state of H_2 to the normal state of H_2^+ produces 1.07 volts of vibrational energy, giving an *apparent* mean ionization potential of 16.25 volts, as measured by electron impact methods. This indirect acquisition of vibrational energy accompanying an electronic transition is now well established, although Condon's numerical value may, of course, be slightly in error.

Some of Richardson's results are given in terms of the new wave me-

chanics, others in terms of the old quantum mechanics. In much of his work on heats of dissociation a value of 15.9 volts has been assumed for I_M, as well as Langmuir's value of 4.21 volts for the heat of dissociation of hydrogen. For these reasons the writer has felt it profitable to work through much of Richardson's data, reducing all the results to a standard basis. The *old* mechanics have been adopted, since this leads to values of the energy levels connected more closely with the observed frequencies. Each electronic level, as given in table 1, thus refers to the molecule with a half unit of vibrational energy, on the interpretation of the new mechanics. Each electronic level of the new mechanics lies beneath that of the old mechanics by the amount

$$s = \omega^0/2 + \omega^0 x/4.$$

The vibration frequency of the new mechanics ($\tilde{\omega}^0$) equals $\omega^0 + \omega^0 x$.

TABLE 1

LEVEL	E_s (CM.$^{-1}$)	DEN.	ω^0 (CM.$^{-1}$)	$\omega^0 x$ (CM.$^{-1}$)	I_0 $\times 10^{40}$	r_0 $\times 10^8$	$I_{\omega\omega^0}$
$1^1S(A)$	zero	0.9396	4262	113.5	0.480	0.76	20500
$2^1S(B)$	90083	1.7920	1325	15.9	1.99	1.55	26400
2^3P	(94735)	(1.9281)	2390	73	0.78	0.97	18700
2^3S	94906.7	1.9337	2593.82	68.41	0.965	1.08	25000
2^1P	(95469)	(1.9526)	.....	....	...	...	...
(C)	99086	2.0882	2380	66.5	0.942	1.06	22400
3^3S	111427	2.9261	.....	....	...	...	...
3^3P	111518.1	2.9365	2306.94	62.94	1.072	1.136	24700
3^1P	111656.8	2.9526	2373.89	12.36(?)	...	...	...
4^3P	117169.9	3.9395	2276.45	63.08	1.089	1.145	24800
4^1P	117216.9	3.9526	2325.6(?)	....	...	...	...
5^3P	119744.2	4.941	2251	58(?)	1.135	1.168	25500
6^3P	121130.2	5.942	2229(?)	54(?)	1.130	1.166	25200
7^3P	121961.0	6.942	.....	....	...	...	...
8^3P	122498.3	7.942	.....	....	...	...	...
H_2^+	124237	∞	2247	61.4	0.927	1.056	20800

The value of $\omega^0 x$ is not changed. The nomenclature is that of the National Research Council Report on Molecular Spectra.[5]

In table 2 are listed the various band systems with the origin (ν_e) of each, from which are derived the energy levels of table 1. In general, the value of ν_e is merely the frequency of the first line of the Q branch, but the error thus introduced is negligible. Richardson's recent conclusions are based entirely on the identification of the lower level of his "A" and "B" bands with Dieke and Hopfield's "B" level (2^1S). The agreement is very striking. Thus the vibrational level intervals, as measured by Richardson are 1313.26, 1276.42 and 1247.67 cm.$^{-1}$, while the same intervals, as measured by Dieke and Hopfield,[6] from the ultra-violet absorption bands, are 1313, 1276 and 1247 cm.$^{-1}$

Richardson considers that the *upper* levels of the A and B bands are 3^1P and 4^1P, and with an assumed Rydberg formula the writer finds

$$\nu = 34{,}154 - 109{,}678.3/(m - 0.0474)^2.$$

The total uncertainty in the "head" of this series should not be greater than about 100 cm.$^{-1}$ Richardson finds a very few lines belonging possibly to bands corresponding to the transition from 5^1P to 2^1S, and uses these also, in a Ritz formula, to get the head of the series at 33,727 cm.$^{-1}$ on the new mechanics, or approximately 34,215 cm.$^{-1}$ on the old. The difference between the Rydberg and Ritz formulas is only 61 cm.$^{-1}$ and, therefore, negligible.

We thus have the term value of the 2^1S level. The normal "A" level lies 90,083 cm.$^{-1}$ below this. Hence, 90,083 + 34,154 = 124,237 cm.$^{-1}$ = 15.34 volts = I_M, on the basis of the old mechanics. With this value established, the remaining values of table 1 follow immediately. Richardson's very reliable value of 29,330.3 cm.$^{-1}$ for the head of the α-β-γ series of bands has been used as the term value of the 2^3S level. The second column of this table gives the frequency of the level measured *up* from the normal

TABLE 2

SYSTEM	TRANSITION	ν_e
A-B (Lyman) bands	1^1S-2^1S	90083
A-C (Werner) bands	1^1S-(?)	99086
"A" bands	2^1S-3^1P	21573.8
"B" bands	2^1S-4^1P	27133.9
.....	2^3P-3^3S	16692.0
"α" bands (Fulcher)	2^3S-3^3P	16611.43
"β" bands	2^3S-4^3P	22263.24
"γ" bands	2^3S-5^3P	24839.05
"δ" bands	2^3S-6^3P	26224.21
"ϵ" bands	2^3S-7^3P	27054.45
"ζ" bands	2^3S-8^3P	27592.52

level, as is customary in the case of molecules, while the third column gives the "denominator" ($\sqrt{109678.3/\nu}$), where ν is the term value measured *down* from ionization, as ordinarily employed in the case of atomic spectra.

The adopted values of I_M = 15.34 volts and D = 4.34 volts (35,154 cm.$^{-1}$) can be checked indirectly by considering the heat of dissociation of the molecule in excited levels. This method has been tried by Richardson[7] and led to the discrepancies noted earlier in this paper. With the new values, the discrepancies disappear. Thus, one may first excite a molecule with electronic energy E_e^x and then dissociate it with vibrational energy D^x (where x is any electronic level). The products of dissociation from an excited molecular level are one normal and one excited atom, according to a considerable amount of experimental evidence which has now accumulated. Hence the total energy just given should equal the energy of dissociation into two normal atoms (D) plus the energy of excitation of one of the atoms (E_A^x), or $E_e^x + D^x = D + E_A^x$.

In order to get D^x, I have used the linear extrapolation of the $\omega^n : n$

curve which Birge and Sponer[8] found so successful in the case of various non-polar molecules. For the 3^3P and 4^3P levels, one thus obtains D^x = 21,120 and 20,520 cm.$^{-1}$, respectively. The dissociation products from these two levels include presumably one hydrogen atom excited to the $3P$ and $4P$ level, respectively. With the known values of $E_A{}^x$ (97,491 and 102,823 cm.$^{-1}$) one obtains, by this indirect process, D^x = 21,127 and 20,807 cm.$^{-1}$, respectively, in perfect agreement with the above directly calculated values. A similar calculation for the 2^3S level gives D^x = 24,580 cm.$^{-1}$ by the direct, and 22,505 cm.$^{-1}$ by the indirect method. This discrepancy of 0.26 volt is more nearly that to be expected, and would vanish for I_M = 15.08 volts. The ω^n:n curves for the ^1P levels are not well enough known for extrapolation. The curve for the 2^1S level has been linearly extrapolated by Dieke and Hopfield with a resulting value of D^x = 3.1 volts. I have, however, recalculated the values of ω^0 and ω^0x, from their data, and have obtained those given in table 1, in place of their values of 1336.7 and 18.42. The new values give 3.41 volts for D^x while the indirect calculation gives 3.37 volts.

For the "C" level we have the recent very reliable data of Hori,[9] which give 99,086 for E_e, and a number of determinations for most of the vibrational level intervals. These are very consistent among themselves but the resulting average values, 2314, 2168, 2050, 1903 and 1783 cm.$^{-1}$ show the curiously oscillating differences 146, 118, 147 and 120 cm.$^{-1}$. The *average* value of $2\omega^0x$ is thus 133 cm.$^{-1}$, but obviously this is very untrustworthy. The resulting value of ω^0 is 2380, as given in table 1. This gives D^x = 2.64 volts, as compared to an indirect measurement of 2.27 volts. Of course, the direct and indirect values of D^x, for the B and C levels, do not throw any light on the value of I_M. They merely test the accuracy of a linear extrapolation of the ω^n curve, and in the case of the B level, where the ω^n values lie on a fairly smooth curve, the agreement is satisfactory.

The numerical values for the respective separations of the vibrational levels of the normal state (1^1S), as measured by Witmer,[10] Dieke and Hopfield[6] and Hori,[9] are essentially the same. The values of ω^0 and ω^0x, as published by these various investigators, are, however, in poor agreement. I have accordingly used the data of all three investigators to obtain the constants given in table 1, which agree most nearly with those given by Witmer. In this connection it should be noted that the separations of the first few vibrational levels lie on a very smooth curve, with the exception of the n = 0 to 1 interval. The best observed value is 4159 cm.$^{-1}$ while the calculated value is 4148 cm.$^{-1}$ This apparent *vibrational perturbation* is similar to that noted by the writer,[11] in the case of the final state of the Ångstrom bands of CO. With the accurate data now becoming available, evidences of such vibrational perturbations are begin-

ning to accumulate and should prove of some theoretical importance.

The values of ω^0 and ω^0x for H_2^+, in table 1, are obtained as follows: Knowing the values of ω^0 and ω^0x for any electronic level, one can obtain s, the shift in this level, on passing to the new mechanics. The corresponding new value of the origin of any band system ($\bar{\nu}_e$) is then given by $\bar{\nu}_e = \nu_e + s'' - s'$. One thus obtains, for the A and B levels, $\bar{\nu}_e = 21{,}056.7$ and 26,647.1 cm.$^{-1}$ Using these values in a Rydberg formula, one finds 33,688 as the new term value ($\bar{E}_e$) of the 2^1S level, compared to Richardson's value of 33,727 cm.$^{-1}$, obtained with a Ritz formula as previously noted. Denoting by s^+ the value of s for the normal level of H_2^+, we have $s^+ = E_e + s - \bar{E}_e$, where E_e, s and $\bar{E}_e$ refer to any one electronic level of H_2. We thus obtain, using the 3^1P and 4^1P levels, $s^+ = 1139$ cm.$^{-1}$ Such a process is essentially equivalent to extrapolating the values of s for a set of levels of H_2 differing only in the value of the total quantum number n to $n = \infty$. In other words, if an electron is removed, step by step, from a molecule, the constants of the molecule should change gradually into those of the molecule ion. This value of $s^+ = 1139$ cm.$^{-1} = 0.141$ volt is in good agreement with Burrau's theoretical value [20] of 0.14 volt. We know also the heat of dissociation for the normal level of H_2^+. It is $D' = 4.34 + 13.54 - 15.34 = 2.54$ volts, or 20,570 cm.$^{-1}$ If one assumes a linear curve for ω^n, then $D' = \omega^{0^2}/4\omega^0x$. Hence, knowing s^+ and D' we obtain ω^0 and ω^0x for this level, as listed in table 1.

In an earlier article, Richardson[12] discussed the band system 2^3P-3^3S of table 2, and concluded that the final state of this system (2^3P) was identical with the "C" level of Dieke and Hopfield. This identification yields $I_M = 15.84$ volts, on the old mechanics, in good agreement with the value expected by Richardson. The identification is based on values of ω^0 and ω^0x for the 2^3P level, of approximately 2390 and 73, as given in table 1. These are to be compared with the corresponding values of 2380 and 66.5, obtained from Hori's new data. Richardson used the older data by Werner[13] for the A-C bands, and a somewhat different interpretation, and obtained a much better check. It is evident from table 1, however, that we cannot simultaneously assume the identification of the C and 2^3P levels, and the B and 2^1S levels. In order to do this, Richardson, in the last part of his recent article,[1] suggests an entire rearrangement of the "B" band system. The writer can see no justification for this rearrangement or for the assumption that the levels C and 2^3P are identical.

The C level is thus left with no theoretical designation. This, however, is no longer a serious objection. Both Richardson[7] and the writer[14] have suggested that the molecule H_2 should be similar to the atom He and should, therefore, have triplet and singlet series. The C level, from its general characteristics, is a ^3P level. It is not the 2^3P level, as we have seen. Still less can it be the 3^3P level. Now Hund,[15] in an extremely

important series of papers, in which the new wave mechanics are applied in some detail to molecular structure, has shown that there are more molecular levels of H_2 than there are atomic levels of He. The C level is, therefore, doubtless one of these extra levels, dissociating into the $n = 2$ state of the hydrogen atom, just as do the known 2^1S, 2^3P and 2^3S levels of H_2.

In all of Richardson's work, and in the discussion given thus far in the present paper, the connection between the ultra-violet and the visible bands of H_2 has been based solely on the spacing of the vibrational energy levels. If the identity of certain levels is correct, there should be corresponding identities for the spacing of the rotational energy levels, as shown by the value of the moment of inertia (I_0). Unfortunately, it has not been possible thus far to obtain trustworthy values of I_0 for the various levels of H_2. The only exception to this statement may be in the case of Hori's recent work. He has obtained new measurements for the A-C (Werner) bands, showing quite surely P, Q and R branches in each band. As usual the "crossing-over" of the Q branch makes it impossible to test the combination principle by means of the Q-P and R-Q relations, but it *is* possible to test it with the R-P relation.[16] Hori thus gets satisfactory values for the spacing of the rotational levels for the normal electronic level, and a value $I_0 = 0.480 \times 10^{-40}$ gm. cm.^{2}, in terms of the *old* mechanics. His published value (0.467) is in terms of the new mechanics, which, as already noted, interprets the former value as referring to a molecule with a half unit of vibrational energy. Since I_n always increases with n, the new mechanics value $\bar{I}_0$ will always be less than the old mechanics value I_0. More explicitly, $\bar{B}_0 = B_0 + \alpha/2$ where $B_n = B_0 - \alpha n$ and $\bar{I}_0 = 27.70 \times 10^{-40}/\bar{B}_0$. Hori obtains $I_0 = 0.942 \times 10^{-40}$ for the C level.

For the A-B (Lyman) bands, the analysis is much less complete. Hori re-analyzes Witmer's published data[10] for the A_n-B_3 progression, obtaining a short R branch and another longer and stronger branch. This latter may be a P, a Q or a combined P and Q branch. Assuming it to be a P branch, he obtains values of ΔF for the A level, agreeing well with those obtained from the Werner bands. The possible test equalities are, however, very few in number. With this interpretation he obtains $B_3 = 12.103$ cm.$^{-1}$ for the B level. This value is at least roughly correct, as may be seen from the following. In the case of a P, Q or R branch, the second difference of the successive lines equals $2C$, where $2C = 2(B' - B'')$. The Werner and Lyman bands are known, from the absorption work of Dieke and Hopfield, to have the same lower level. Hence the value of B'' for the Lyman bands necessarily equals that for the Werner bands. Thus, *regardless* of the interpretation of the observed branches in the Lyman bands, we obtain a value of $2C$, giving in turn a value of B_3 for the upper level of the Lyman bands. The data show that this value (for the $n = 3$ level) is certainly close to 12.1 cm.$^{-1}$

One can obtain an approximate value of B_0 for the $n = 0$ level as follows: For the A level, $\alpha = 2.7$, according to Hori's work, and $B_0 = 57.77$. Hence, $\alpha/B_0 = 0.047$. For the C level $\alpha = 1.2$ and $B_0 = 29.4$. Hence, $\alpha/B_0 = 0.041$. The near equality of these two ratios seems significant. If then we assume $\alpha/B_0 = 0.044$ for the B level, we have $B_3 = 12.10 = B_0 - 3\alpha = B_0(1 - 0.132)$. Hence, $B_0 = 13.94$ cm.$^{-1}$ or $I_0 = 1.99 \times 10^{-40}$ gm. cm.^{2} for the B level. This is a surprisingly large value, especially as compared to the value of I_0 for H_2^+. That the relative values of I_0 for the A, B and C levels are, however, reasonable can be seen by comparing the values of $I_0\omega^0$ as given in the last column of table 1. This product has been found to be roughly constant, for the various levels of other molecules.

Richardson has not as yet been able to apply the combination principle in the case of any of the visible bands of hydrogen. He has calculated rough values of the moments of inertia by using certain theoretical relations between vibrational and rotational energy constants,[17] together with certain additional assumptions. Some of the theoretical relations just mentioned have been found true in the case of rather rigid molecules having long band series, but it seems to the writer questionable how far they can be trusted in the case of a very deformable molecule like hydrogen, especially with such fragmentary data available. Richardson's value of $2B_0$ (= his $u\omega^0$) for the 2^1S level is 55.7 and is especially uncertain. It, however, is almost exactly twice the value of $2B_0 = 27.88$, just obtained for the B level, which is certainly identical with the 2^1S level. It may be noted that Richardson's value of $2B$ is practically the same for the other excited levels, although the value of ω^0 is twice as large. In other words, the value of $I_0\omega^0$ is roughly the same for all the levels of H_2, as shown in table 1, but if Richardson's value of $2B_0$ for the B level is used, this product is only 13,200. It is, therefore, possible that in some way a factor of two has been introduced into Richardson's analysis of this level. For the 2^3P level Richardson obtains $I_0 = 0.78 \times 10^{-40}$, while for the C level Hori obtains 0.94×10^{-40}. This discrepancy would appear to rule out the suggested identity of these two states, but it must be remembered that Richardson's value is very uncertain. Richardson's values of I_0 for various other levels, together with all values of the nuclear separation r_0[18] are given in table 1.

The above analysis thus indicates that Richardson's identification of the B level of Dieke and Hopfield with the final level of his A and B bands is justified by the vibrational level intervals, and by the direct and indirect evaluation of the various heats of dissociation. It is justified by the rotational level intervals only if Richardson's value of $2B_0$ is halved, giving $I_0 = 1.99 \times 10^{-40}$ for this common level, in agreement with Hori's analysis. The identification of the 2^3P and C levels is inconsistent with

the previous identification, while the expected equalities among the rotational and vibrational energy levels are not conclusive for or against this assumption.

Assuming then the first identification, we have the relations given in tables 1 and 2 leading to $I_M = 15.34$ volts, with $D = 4.34$ volts assumed for the neutral molecule and $I'_M = 16.08$ volts $(4.34 + 2 \times 13.54 - 15.34)$, and $D' = 2.54$ volts for the ionized molecule, all in terms of the *old* mechanics. In terms of the new wave mechanics $I'_M = 16.22$ volts, as compared to Burrau's theoretical value of 16.30 volts (or to a value of 16.46 volts recently calculated in a somewhat similar way by Niessen[19]) while $I_M = 15.46$ volts. Hence, $I'_M + I_M = 31.68$ volts, as compared to Condon's approximate theoretical value of 31.48 volts. Finally, if Burrau's theoretical value of $I'_m = 16.30 \pm 0.03$ volt is assumed to be correct, then $D = 4.42$ volts, instead of 4.34 volts as assumed throughout this paper.

[1] Richardson, *Proc. Roy. Soc.*, **A115,** 528, 1927.

[2] Burrau, *Naturwiss.*, **15,** 16, 1927, and *Kong. Danske Vid. Selsk. Math.-fys.* **7,** No. 14, 1927.

[3] The most probable values of the basic constants e, c and h now are $e = 4.774 \times 10^{-10}$ es., $c = 2.99796 \times 10^{10}$ cm. sec.$^{-1}$, $h = 6.557 \times 10^{-27}$ erg sec., yielding 8100 cm.$^{-1}$ = one volt.

[4] Condon, *Proc. Nat. Acad. Sci.*, **13,** 466, 1927.

[5] Bull. Nat. Res. Coun., Vol. 11, Part 3, 1926, "Molecular Spectra in Gases." To be referred to as "Report."

[6] Dieke and Hopfield, *Z. Physik*, **40,** 299, 1926, and *Phys. Rev.*, **30,** 400, 1927.

[7] Richardson, *Proc. Roy. Soc.*, **A113,** 368, 1926 (see page 415), and *Ibid.*, **A114,** 643, 1927 (see page 649).

[8] Birge and Sponer, *Phys. Rev.*, **28,** 259, 1926.

[9] Hori, *Z. Physik*, **44,** 834, 1927.

[10] Witmer, *Phys. Rev.*, **28,** 1223, 1926.

[11] Birge, *Ibid.*, **28,** 1157, 1926.

[12] Richardson, *Proc. Roy. Soc.*, **A114,** 643, 1927.

[13] Werner, *Ibid.*, **A113,** 107, 1926.

[14] "Report," page 229.

[15] Hund, *Z. Physik*, **40,** 742, 1927, and **42,** 93, 1927.

[16] "Report," page 147.

[17] *Ibid.*, page 108.

[18] *Ibid.*, page 223. For H_2, $\mu = 0.8314 \times 10^{-24}$. The values of I_0 and r_0 for H_2^+ are from Burrau.[2] (r_0 = two Bohr units.)

[19] Niessen, *Z. Physik*, **43,** 694, 1927.

[20] Dr. E. U. Condon has kindly called the writer's attention to the fact that on page 13 of Burrau's Danish Academy paper, the equation should read $2/M$ in place of $1/2M$. This gives $s^+ = 0.14$ volt, instead of Burrau's published value of 0.07 volt.

THE SCATTERING OF X-RAYS FROM GASES

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The angular distribution of intensity of X-rays scattered from amorphous substances is affected by two types of diffraction phenomena: interference between rays scattered by groups of molecules, and interference between rays from groups of electrons within the atoms. It has been repeatedly suggested[1] that experiments with amorphous substances may lead to definite information about the distribution of electrons in atoms through this latter effect. Debye[2] has shown that in an ideal gas the effect due to neighboring molecules is negligible under ordinary conditions. For studying electron distribution, therefore, the ideal gases are considered most desirable as scattering substances, and that the problem may be simple the light gases have been recommended. Mark and Schocken[3] have recently obtained the intensity distribution of Mo K_α rays scattered from carbon dioxide and argon. With the latter gas they find no interference effect between 30° and 160°; with carbon dioxide they observe none between 40° and 140°, but their 30° observation shows a small excess over the $(1 + \cos^2\theta)$ law of scattering of electrons at rest scattering independently. Experiments with hydrogen or helium have long been wanted. If the classical electrodynamics are applicable they should serve to locate the pair of electrons in the molecule, while if the Compton effect is very prominent they should show almost no interference effects.

The apparatus used in the present experiment is drawn to scale in figure 1. A water-cooled molybdenum target X-ray tube was operated at 55,000 peak volts, 60 milliamperes, while connected to mercury pumps. The radiation was passed through filters at F, through slit S_1 and entered chamber C through a window of thin celluloid. The pressure of a gas contained in C could be varied from 0.5 to 335 cms. Secondary radiation from a small volume of this gas passed through slits S_2 and S_3 into an ionization chamber. The ionization current at each angle θ was measured by the rate of deflection of a Compton electrometer, and was found to vary linearly with the pressure of the gas in C.

Commercial electrolytic hydrogen (containing 0.03% oxygen), oxygen (containing 0.1% impurity) and commercial carbon dioxide were admitted to the chamber C through drying tubes and a tube of glass wool. Electrometer readings with the gases at 0.5 cm. pressure were subtracted from the readings at 335 cms. at each angle. To determine the volume of the gas effective in causing ionization at the different angles, the path of the primary beam in C was filled with loose cotton, and the ionization thus

obtained was compared with that from a small string of the same cotton, which gave a constant volume at all angles. The intensity curves for the different gases are more accurate relative to each other than relative to the cotton, which is only approximately homogeneous, so the conclusions drawn from the curves are based more on the ratios of their ordinates than on the absolute values of them.

Monochromatic radiation was obtained by a filter method recently used by Prof. P. A. Ross. Filters of strontium oxide and zirconium oxide were adjusted to such thickness that each absorbed 50% of the Mo K_β line. They then were found well matched at all wave-lengths except those between their K absorption limits. Placing first one then the other in the primary beam and taking the difference of the intensity transmitted,

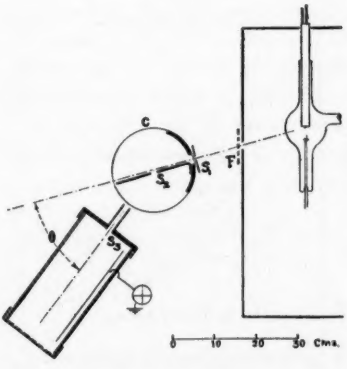

FIGURE 1
Diagram of apparatus.

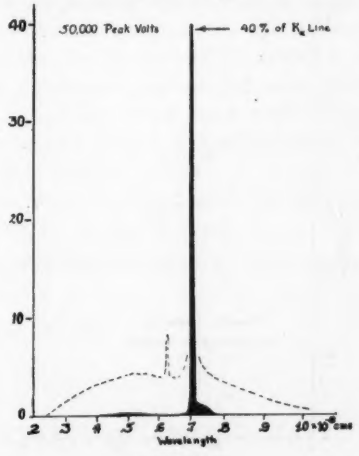

FIGURE 2
Spectrum of X rays used.

a spectrum shown by the black area in figure 2 was obtained. (The dotted curve is the spectrum with no filter in the beam.) Considering the cross-sectional area of this monochromatic beam compared with that obtained with a crystal reflected beam, the energy used in these experiments was at least several hundreds of times greater than obtained by the usual crystal method.

Figure 3 gives the weighted mean of measurements with hydrogen as the scatterer. Within a probable experimental error of 5% the intensities using 0.71 Å monochromatic radiation (circles) do not differ from the intensities obtained with one filter in position (dots), though the spectrum of the latter resembles the dotted curve of figure 2, and has a mean effective wave-length less than 0.71 Å. No interference effect, which would be a

function of the wave-length, is noticeable. If the two electrons of a hydrogen molecule are assumed to be a constant distance of 1.1 Å apart, as postulated in an old form of Bohr's theory, the classical theory gives the dotted curve of figure 3. The intensity on this theory at θ degrees from the direct beam is proportional to

$$(1 + \cos^2\theta)\left[1 + \frac{\sin\left(\frac{4\pi s}{\lambda}\sin\frac{\theta}{2}\right)}{\frac{4\pi s}{\lambda}\sin\frac{\theta}{2}}\right]$$

where s is the electron separation and λ the wave-length of the rays.

On the other hand, if the radiation scattered at these angles were mostly

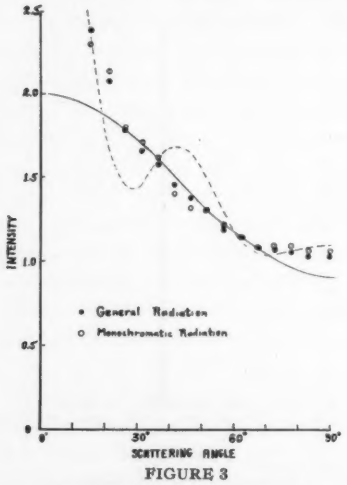

FIGURE 3
Scattering from hydrogen gas.

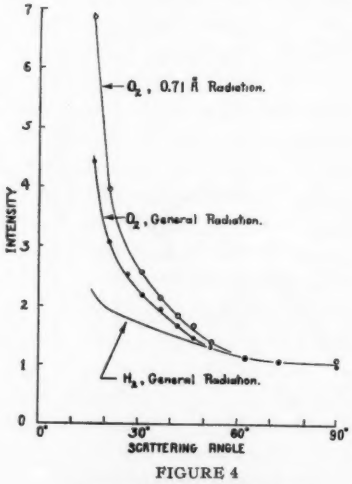

FIGURE 4
Scattering from oxygen gas.

modified, the intensity should be practically independent of the wave-length used, as is observed here. The intensity formulas of Compton, Breit, Dirac (Heisenberg-Born mechanics) and Gordon (Shroedinger theory) agree approximately in predicting the intensity proportional to

$$\frac{1 + \cos^2\theta}{\left(1 + \frac{h\nu}{mc^2}\,\mathrm{vers}\,\theta\right)^3}.$$

This formula, for the frequency used, is plotted as the solid curve in figure 3. The deviations from it that are observed may be due to errors in determining the volume of gas scattering, or possibly the unmodified radiation

becomes appreciable at small angles and causes excess scattering. The points at small angles, however, cannot be taken as decisive evidence of the latter because at those angles the scattering from the walls of the chamber was larger than at greater angles and the accuracy of the gas readings consequently lower.

In figure 4 it is seen that the oxygen curve is a function of the wave-length. Reduced to the same scale as the hydrogen curve at 70° it shows that excess scattering for θ less than about 60° is very prominent. A few observations with carbon dioxide gave a curve hardly distinguishable from that for oxygen; excess scattering appeared at approximately the same angle. Work on other gases is in progress.

One may conclude on the basis of these experiments that interference is present in X-rays scattered from a single molecule of carbon dioxide or oxygen, and absent in rays scattered from a hydrogen molecule—at least in the range 30° to 90°. This may be interpreted as evidence of considerably unmodified radiation in the former case and of completely or nearly completely modified radiation in the latter, confirming Woo's measurements and in accord with Jauncey's theory of the intensities of the different kinds of radiation in the Compton effect.

It is a pleasure to acknowledge the many helpful suggestions of Prof. A. H. Compton, Prof. P. A. Ross and Dr. J. A. Bearden.

[1] Compton's *X-Rays and Electrons*, p. 74, gives complete references and a summary of the work in this field.

[2] Debye, P., *Phys. Zeit.*, **28**, 135, 1927.

[3] Mark and Schocken, *Naturwiss.*, **15**, 139, 1927 (Feb. 11).

THE DISTRIBUTION OF ENERGY IN MOLECULES

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The author has recently attempted to account for the rates of unimolecular reactions by a theory[1] which assumes activation by collision and treats the reacting molecules as mechanical systems with many internal degrees of freedom. A somewhat similar theory had been advanced earlier by Rice and Ramsperger.[2]

For the complete development of the author's theory it is necessary to solve the problem: Given a number of oscillators, classical or quantum, harmonic or otherwise, and a value for the total energy of these oscillators, calculate the chance that a specified one of the oscillators shall have energy in excess of some given value. The solution of this problem for the case

of classical harmonic oscillators was given in the author's first paper. The methods used there, however, were somewhat elementary and not suitable for more complex cases.

The problem may be solved, in general, by a consideration of the total phase-space volumes associated with various conditions of the molecule. Some of the special cases have been worked out, and will be considered here.

Case I—Classical Oscillators.—Suppose that there are s classical oscillators, and that the energy of the ith oscillator is given by

$$\epsilon_i = p_i^2 + q_i^2.$$

The coördinates may always be transformed so that this form is obtained, rather than

$$\epsilon_i = \tfrac{1}{2}b_i p_i'^2 + \tfrac{1}{2}a_i q_i'^2.$$

Then, from the well-known formula[3] for the volume of an n-dimensional sphere, the phase-space volume corresponding to molecules with energy between ϵ and $\epsilon + d\epsilon$ is

$$\int_{\sum_1^s (p_i^2 + q_i^2) = \epsilon}^{\epsilon+d\epsilon} \cdots \int dp_1 \ldots dp_s dq_1 \ldots dq_s = \frac{\pi^s}{\Gamma(s+1)} [(\epsilon + d\epsilon)^s - \epsilon^s]$$

$$= \frac{\pi^s}{\Gamma(s)} \epsilon^{s-1} d\epsilon.$$

The volume corresponding to states in which the total energy lies within the limits ϵ and $\epsilon + d\epsilon$ and in which a specified oscillator has energy at least ϵ_0 is[4]

$$\int_{p_1^2+q_1^2=\epsilon_0}^{\epsilon} \int dp_1 dq_1 \int_{\sum_2^s (p_i^2 + q_i^2) = \epsilon - p_1^2 - q_1^2}^{\epsilon - p_1^2 - q_1^2 + d\epsilon} \cdots \int dp_2 \ldots dp_s \, dq_2 \ldots dq_s$$

$$= \int_{p_1^2+q_1^2=\epsilon_0}^{\epsilon} \int \frac{\pi^{s-1}}{\Gamma(s-1)} (\epsilon - p_1^2 - q_1^2)^{s-2} \, d\epsilon dp_1 dq_1.$$

This may be conveniently integrated by the substitutions

$$p_1 = \sqrt{z} \cos\theta,$$

$$q_1 = \sqrt{z} \sin\theta,$$

where, by Jacobi's theorem, we must substitute

$$dp_1 dq_1 = \tfrac{1}{2} dz d\theta.$$

The result is

$$4\int_{\epsilon_0}^{\epsilon}\int_0^{\pi/2} \frac{\pi^{s-1}}{\Gamma(s-1)}(\epsilon-\epsilon_0)^{s-2}\cdot\tfrac{1}{2}d\epsilon d\theta d\epsilon_0 = \frac{\pi^s}{\Gamma(s)}(\epsilon-\epsilon_0)^{s-1}\,d\epsilon.$$

The ratio of this volume to the total volume associated with energy ϵ to $\epsilon + d\epsilon$, namely

$$\left(\frac{\epsilon-\epsilon_0}{\epsilon}\right)^{s-1},$$

is the chance that when the s oscillators have energy between ϵ and $\epsilon + d\epsilon$ a specified one will have energy greater than ϵ_0.

Case II—Quantum Oscillators of a Single Frequency.—Suppose that there are s quantum oscillators all of the frequency ν. Let the group have n quanta; calculate the chance that some one of the oscillators has at least m quanta.

We shall assume that the μ weight for each microscopic state of the system is h^s, and that there is no difficulty due to a greater μ weight for the lowest state. Then the total μ weight associated with the system when it contains n quanta is

$$h^s\cdot\frac{(n+s-1)!}{n!(s-1)!} = h^s\binom{n+s-1}{n}$$

since the number of distinct arrangements of n objects into s groups is

$$\binom{n+s-1}{n}.$$

The μ weight of those states in which a chosen oscillator has at least m quanta is[5]

$$\sum_{m=m}^{n} h\cdot h^{s-1}\binom{n-m+s-2}{n-m}.$$

Upon writing $n - m = p$, this becomes

$$h^s\sum_{p=0}^{p}\binom{p+s-2}{p},$$

and it is well known that the value of this sum is[6]

$$h^s\binom{p+s-1}{p}.$$

Hence, the μ weight of those states in which the chosen oscillator has at least m quanta is

$$h^s \binom{n-m+s-1}{n-m}$$

and the ratio of this to the total μ weight gives the chance that when the s oscillators have n quanta, a chosen one will have at least m quanta. This is

$$\frac{n!(n-m+s-1)!}{(n-m)!(n+s-1)!}.$$

It is very easy to show that when both n and $n-m$ are very large this reduces to the classical form,

$$\left(\frac{\epsilon-\epsilon_0}{\epsilon}\right)^{s-1},$$

as of course it must.

Case III—Classical Oscillators and Quantum Oscillators of a Single Frequency.—(*A.*) Accumulation of energy in a quantum oscillator. Suppose that there are s classical oscillators, and r quantum oscillators, all of frequency ν. Then the total μ weight corresponding to energy between ϵ and $\epsilon + d\epsilon$ is[7]

$$\sum_{q=0}^{n} h^r \binom{q+r-1}{q} \int\cdots\int_{\sum_1^s (p_i^2+q_i^2)\,\epsilon - qh\nu}^{\epsilon-qh\nu+d\epsilon} dq_1 \ldots dq_s\, dp_1 \ldots dp_s$$

$$= \frac{\pi^s}{\Gamma(s)} h^r d\epsilon \sum_{q=0}^{n} \binom{q+r-1}{q} (\epsilon - qh\nu)^{s-1}.$$

Here n is the greatest number of quanta which an oscillator can have; it is defined by the inequality

$$n \leqq \frac{\epsilon}{h\nu} < n+1.$$

Now the μ weight of the states in which the total energy is between ϵ and $\epsilon + d\epsilon$ and in which a chosen quantum oscillator has at least m quanta is[8]

$$\sum_{p=m}^{n} h \cdot \frac{\pi^s}{\Gamma(s)} h^{r-1} d\epsilon \sum_{q=p}^{n} \binom{q-p+r-2}{q-p} (\epsilon - qh\nu)^{s-1}.$$

Note that

$$\sum_{p=m}^{n} \sum_{q=p}^{n} f(q,p) = \sum_{q=m}^{n} \sum_{p=m}^{q} f(q,p).$$

This may be seen by writing out the sum in a square array. Using this result, we get

$$\frac{\pi^s}{\Gamma(s)} h^r d\epsilon \sum_{q=m}^{n} (\epsilon - qh\nu)^{s-1} \sum_{p=m}^{q} \binom{q-p+r-2}{q-p}$$

$$= \frac{\pi^s}{\Gamma(s)} h^r d\epsilon \sum_{q=m}^{n} \binom{q-m+r-1}{q-m} (\epsilon - qh\nu)^{s-1}.$$

The ratio of this to the total μ weight gives the chance that the quantum oscillator will have at least m quanta. This is

$$\frac{\sum_{q=m}^{n} \binom{q-m+r-1}{q-m} (\epsilon - qh\nu)^{s-1}}{\sum_{q=0}^{n} \binom{q+r-1}{q} (\epsilon - qh\nu)^{s-1}}.$$

(*B.*) Accumulation of energy in a classical oscillator. Let there be s classical oscillators, and r quantum oscillators of frequency ν, just as before. The total μ weight of the states in which the energy of the system is between ϵ and $\epsilon + d\epsilon$ is the same as before. The μ weight of those states in which a chosen classical oscillator has energy at least ϵ_0 is[9]

$$\frac{\pi^s}{\Gamma(s)} h^r d\epsilon \sum_{p=0}^{n-m} \binom{p+r-1}{p} (\epsilon - ph\nu)^{s-1}$$

where n and m are defined by the inequalities

$$n \leqq \frac{\epsilon}{h\nu} < n + 1,$$

$$n - m \leqq \frac{\epsilon - \epsilon_0}{h\nu} < n - m + 1.$$

The ratio gives the chance we want, namely

$$\frac{\sum_{p=0}^{n-m} \binom{p+r-1}{p} (\epsilon - ph\nu - \epsilon_0)^{s-1}}{\sum_{p=0}^{n} \binom{p+r-1}{p} (\epsilon - ph\nu)^{s-1}}.$$

This may be put in the alternative form

$$\frac{\sum_{q=m}^{n} \binom{q-m+r-1}{q-m} (\epsilon - qh\nu + mh\nu - \epsilon_0)^{s-1}}{\sum_{q=0}^{n} \binom{q+r-1}{q} (\epsilon - qh\nu)^{s-1}},$$

which is very similar to that for accumulation of energy in a quantum oscillator since, in general, $mh\nu$ is about equal to ϵ_0. For the classical case it is not possible to define m by a single equation, but the notation preserves the formal identity between the two cases as much as possible. Indeed, when ϵ_0 is an integral number of quanta, then $mh\nu = \epsilon_0$, and the two chances become identical. This gives us the very important result:

In a group of oscillators, some of which are classical and some quantum, all the quantum oscillators having the same frequency, the chance that a given classical oscillator shall have energy equal to or greater than that of m quanta, is exactly the same as the chance that a given quantum oscillator shall have m or more quanta. It is, of course, necessary that the entire group be in statistical equilibrium.

Thus, when the quantum frequencies are not too high, or the temperature of the system too low, the distribution of energy among the oscillators is not markedly different for the quantum oscillators from that for the classical. It will, however, usually be very much different from that which would exist if all the oscillators were classical or all quantum. This difference will be most marked when we consider the chance that an oscillator shall have a large share of the total energy.

This is illustrated by the following short table, constructed for five classical oscillators and five quantum oscillators, with a total energy equivalent to 20 quanta. The second column gives the chance that an oscillator of this group shall have energy equivalent to at least the number of quanta given in the first column. The third column gives the corresponding chance if all the oscillators were classical, and the fourth if they were all quantum.

TABLE 1

m	MIXED CHANCE	PURE CLASSICAL CHANCE	PURE QUANTUM CHANCE
0	1.000	1.000	1.000
5	0.102	0.751×10^{-1}	0.131
10	0.471×10^{-3}	0.195×10^{-2}	0.922×10^{-2}
15	0.395×10^{-4}	0.381×10^{-5}	0.200×10^{-3}
19	0.105×10^{-7}	0.195×10^{-11}	0.998×10^{-6}
20	0	0	0.998×10^{-7}

Thus it appears that the tendency for energy to accumulate in a single oscillator, other conditions being equal, is much greater for quantum oscillators than for classical, when all the oscillators are of one kind; when they are of both kinds, the chance becomes much more nearly the same for the two kinds, and lies between the two pure chances.

Case IV—Classical Oscillators and Quantum Oscillators of Two Frequencies.—It is not hard to extend the preceding results to the case of quantum oscillators of two frequencies and classical oscillators, although

the formulas become more complex. The result for accumulation of energy in a classical oscillator will be given here. The others are very similar.

Suppose that there are s classical oscillators, r quantum oscillators of frequency ν, and t quantum oscillators of frequency μ. Let the entire system have energy between ϵ and $\epsilon + d\epsilon$. Then the chance that a single selected classical oscillator will have energy at least ϵ_0 is found to be

$$\frac{\sum_{p=0}^{k}\sum_{q=0}^{l(p)}\binom{p+r-1}{p}\binom{q+t-1}{q}(\epsilon - ph\nu - qh\mu - \epsilon_0)^{s-1}}{\sum_{p=0}^{n}\sum_{q=0}^{l'(p)}\binom{p+r-1}{p}\binom{q+t-1}{q}(\epsilon - ph\nu - qh\mu)^{s-1}}$$

where

$$n \leqq \frac{\epsilon}{h\nu} < n + 1,$$

$$k \leqq \frac{\epsilon - \epsilon_0}{h\nu} < k + 1,$$

$$l(p) \leqq \frac{\epsilon - \epsilon_0 - ph\nu}{h\mu} < l(p) + 1,$$

$$l'(p) \leqq \frac{\epsilon - ph\nu}{h\mu} < l'(p) + 1.$$

It is fairly evident how to extend this formula to still more complex cases, merely by analogy, though the specification of the upper range of summation becomes increasingly complicated.

The application of these results to the theory of unimolecular reactions will be considered in another place, but it may be said here that, while it would be possible to account for the decomposition of nitrogen pentoxide, using 15 classical internal degrees of freedom (this is nearly the maximum possible for a mechanical system of seven mass-points), when these degrees of freedom are quantized, with frequencies so selected as to give reasonable specific heat values, it is no longer possible to account for the observed rates by the author's theory. Furthermore, it is evident from the results which have been given that no combination of classical and quantum oscillators could be much more effective in accounting for these rates than any other combination of the same number of oscillators with the same internal energy at the temperatures of interest.

These results are all for harmonic oscillators. It will be shown in a subsequent article, however, that there is no help to be obtained by the introduction of a special type of anharmonic oscillator, which has a certain formal resemblance to a chemical bond.

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[1] Kassel, L., article to appear soon in *J. Phys. Chem.*

[2] Rice, O. K., and Ramsperger, H. C., *J. Amer. Chem. Soc.*, **49**, 1617 (1927).

[3] See, for example, Tolman, *Statistical Mechanics with Application to Physics and Chemistry*, Chemical Cat. Co., New York, **1927**, p. 129.

[4] This evidently gives only that part of the preceding volume for which $p_1^2 + q_1^2$ is at least equal to ϵ_0. The same sort of integration will be made repeatedly in this paper.

[5] The μ weight for any state of a single oscillator is h; we extend the summation from $m = m$ to $m = n$, since we want all states in which the single oscillator has at least m quanta.

[6] See, for example, Laska, W., *Sammlung von Formeln der reinen und angewandten Mathematik*, Vieweg und Sohn, Braunschweig, p. 15. But the proof is very easily given by induction.

[7] The integral gives the contribution to the product of the classical oscillators; this is multiplied by the quantum contribution, and the sum taken over all possible total energies for the quantum oscillators.

[8] The expression following the first h is the total μ weight for s classical oscillators and $r - 1$ quantum oscillators, with total energy between $\epsilon - qh\nu$ and $\epsilon - qh\nu + d\epsilon$.

[9] To get this, we multiply the contribution of r quantum oscillators with p quanta by that of s classical oscillators with energy between $\epsilon - ph\nu$ and $\epsilon - ph\nu + d\epsilon$ and at least ϵ_0 in a single oscillator, and sum over all possible values of p.

THE LIFE OF ATOMIC STATES AND THE INTENSITY OF SPECTRAL LINES

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It has recently been possible to explain most of the strong nebular lines as lines arising from electron jumps from metastable states in oxygen and nitrogen.[1] In the nebulae the emission of spectral lines can be observed under conditions of much higher rarefaction than can be found in any other terrestrial or astronomical source. Since these nebular lines occur only under these conditions, this very definitely indicates that the electron jumps causing them take place spontaneously without the interference of outside fields due to other ions, that can often be used to explain jumps of this sort when they are observed in terrestrial sources. *This constitutes then the first direct evidence that metastable states are not absolutely metastable but are states of long mean life, i.e., states from which the probability of a spontaneous jump in unit time is very small.*

Further evidence for this viewpoint is found in the observation by Rayleigh[2] that the forbidden mercury line ^1S-$^3P_2$ at 2270 Å is absorbed about 1/1,000,000 as strongly as the ^1S-$^3P_1$ line at 2536 Å. Since there

is a slight probability of absorption there must also be a slight probability of emission as well. In this experiment no ions were present although, since the pressure was high, the field due to the electric dipoles in other atoms might have been appreciable.

As was pointed out in the discussion of the nebular spectrum, this viewpoint makes it possible to explain the enormous variation in intensity of the nebular lines in going from terrestrial sources, where the lines are too weak to be observed, to the nebulae, where they are often the strongest lines in the spectrum. The intensity of the line depends directly on the ratio of the probability of the return of an atom from a higher state to a lower state with emission of that line to the sum of the probabilities of its return by all processes combined. Since the mean life of any given process is usually defined as the reciprocal of its probability in unit time, it is evident that the intensity of the line depends primarily on the ratio of the mean time between collisions of the second kind with other atoms or with the wall to the mean life of the state before spontaneous emission. If this ratio is small the majority of the atoms are taken out of the excited state by collisions of the second kind rather than by the emission of radiation and the line is weak. If, on the other hand, the ratio is large the atoms can radiate spontaneously without interference and the line appears strongly.

These same differences in lifetime can doubtless be used to explain various anomalous intensities found, to a lesser degree, in terrestrial sources. For example, calculations on the basis of the correspondence principle show that the intercombination jumps, i.e., jumps between a singlet and a triplet term, etc., in the simplest atoms are very improbable, or, interpreted on the above basis, have a very long life. In the case of the heavier atoms such as mercury the perturbations due to the other electrons are so great that the intercombination jumps have nearly the same probability per unit time as the regular jumps. The 2536 Å line in mercury which is caused by an intercombination jump is one of the strongest lines. Even for this line, however, Wien[3] finds a mean life before emission nearly five times as long as the life of the other mercury lines.

On going to the lighter elements such as magnesium, however, the intercombination line ^1S-$^3P_1$ at 4571 Å is found to be quite weak even though it is the true resonance line of neutral magnesium. This is at once explicable if the low probability of this jump in unit time is interpreted as indicating a mean life so long that the majority of the atoms are returned by collisions of the second kind rather than by radiation.

In beryllium the corresponding line has never been observed although it has been carefully searched for under low-temperature furnace conditions that are known to be especially favorable for bringing out these resonance lines.[4] This can be explained as due to a still lower probability

per unit time and longer life, even comparable perhaps with that of the nebular lines.

This indicates a whole gradation of mean lives extending from that of the regular lines whose mean life before emission is of the order of 10^{-8} seconds to that of the nebular lines whose mean life before emission must be of the order of a second or more.

[1] Bowen, *Nature*, **120**, 473, 1927; *Pub. A. S. P.*, **39**, 295, 1927.
[2] Rayleigh, *Nature*, **120**, 295, 1927.
[3] Wien, *Ann. Phys.*, **73**, 483, 1924.
[4] A. S. King, unpublished results.

THE SPECTRA OF SINGLY AND DOUBLY IONIZED GERMANIUM (GE II AND III)

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In a paper appearing in 1924, Lunt[1] has given the measurements and intensities of the germanium lines appearing in the spectrum from a discharge tube containing Ge Cl_4 and also the metallic spark in air from 3937 Å to 7147 Å. The measures of Exner and Haschek from 4744 Å to about 2200 Å for the arc and spark are also available in Kayser's "Handbuch de Spectroscopie," Vol. V. Lunt has also placed the lines in three classes. In class I are those arc and spark lines which are not suppressed by self-induction; in class II are lines which are strong when self-induction is used in the metal spark in air but vanish completely when self-induction is eliminated, while in class III are given what the author calls "enhanced" lines. Lunt also gives a list of lines of common frequency separations in which, among others, is found three pairs having the common separation 360 and placed in class I.

In connection with the work here reported the metallic spark has been photographed by the author on the two-meter grating from 2000 Å to 4000 Å in air, from 1250 Å to 2400 Å in hydrogen and from 400 Å to 4000 Å in vacuum. The grating used was ruled, however, for the Lyman region and gives nothing but the stronger lines in the longer wave-lengths above about 3000 Å. For the intensity in the vac spark of certain lines longer than 4000 Å and for the wave-length of those (see * Table 2) the author is indebted to Prof. Stanley Smith of this laboratory.

I. *The Spectrum of Singly Ionized Germanium.*—The ordinary series spectrum of Ge II will consist, according to the displacement law, of doub-

lets, the separations of which can now be fairly accurately estimated either from those of Ga I[2] or Si II.[3] Also it seemed likely that the doublet $4P$-$5S$ by comparison with Si II and Pb II[4] would fall between 1500 Å and 1900 Å. By photographing the spark in hydrogen a persistent doublet appears near 1600 Å with strong intensity and giving $\Delta\gamma = 1769$. Two independent estimates of this separation have been made. McLennan and McLay,[5] in their work on the arc spectrum, estimate it to be greater than $3\overline{P}_2$-$3\overline{P}_1$ which has the value 1415.7 in Ge I. Green and Loring,[6] whose paper on Sn appeared during the progress of this work, make an estimate from the ^2P difference from the arc spectrum of Ge of the value 1900.

The $5P$ separation was also given by Green and Loring as 360 from the principal pair at 5895 Å and 6022 Å, thus confirming the choice already made by the author for this doublet. Thus it seemed likely that the other pairs having this separation which were listed by Lunt belong to the $5P$ levels of Ge II. The $5S$ term was now estimated to be close to 67,000 and from this the other terms were calculated and most of the groups in table 1 were located. After the $4P$-$6S$ doublet had been found it was possible to calculate the $4P$ terms and the values obtained by this means are those given in table 1 and all the other term values have been made to conform to these. The wave-lengths of all lines are reduced to I. A. in vacuum.

It will be seen that all of the terms are regular with the exception of $5D$ which does not seem to fit into the scheme. It was also found that the value of the $4P$ terms calculated by means of the lines in the $4P$-$4D$ and $4P$-$5D$ groups does not correspond well to the value obtained from the S combinations as described above.

The ordinary terms such as are discussed above all converge to the singlet S term of Ge III the approximate value of which is 259,068 cm.$^{-1}$ The lines found corresponding to combinations of these terms are almost all first members of the well-known series and all arise from the single $4p$ electron being shifted to $4d$, $4f$, $5s$, etc., orbits while the two $4s$ electrons remain in the $4s$ orbits. But when one of these $4s$ electrons is disturbed so that it also moves in a $4p$ orbit then other terms arise of the type ^2S, ^2D, $^2\overline{P}$ and $^4\overline{P}$, according to the theory of Hund. Two of these have been found, the $^2\overline{P}$ term combining with $4P$ to give a four-line multiplet near 1100 Å and the ^2D term being that labelled pD in table 1. The latter combines with P and F terms just as it does in Sn II. The two remaining terms have not been located for certain. Many pairs of lines occur in the region below 2000 Å having separations about that for $4P$ but the measurements above 4000 Å are too scanty to make much progress.

Further confirmation of the identification of these groups in Ge II was sought in the spectrum of As III by the use of Moseley lines employing

the values for Ga I from Fowler's report. All the main groups in As have been located in this way and a further report will be made on the spectrum.

TABLE 1

SERIES SPECTRUM OF Ge II

DESIGNATION	λ(I. A. VAC.)	INT.	ν	Δν	TERM VALUES		R. D.
pD_2-$5P_2$	6969.4		14348	170	$5S$	67026	2.5591
pD_3-$5P_2$	7052.9		14178	360			
pD_2 $5P_1$	7148.7		13988		$6S$	34648	3.5592
$5P_2$-$6S$	6485.95	5	15418	360			
$5P_1$-$6S$	6337.91	3	15778				
					$4P_1$	129428	1.8416
$5S$-$5P_2$	5895.01	10	16963.5	359.6	$4P_2$	127660	1.8543
$5S$-$5P_1$	6022.67	8	16603.9				
$5P_2$-$5D_2$	4825.78	3	20722	42	$5P_1$	50423	2.9505
$5P_2$ $5D_3$	4816.04	9	20764	361	$5P_2$	50063	2.9611
$5P_1$-$5D_2$	4743.17	8	21083				
$4D_3$-$4F$	5179.98	7	19305.1	176.2	pD_2	64415	
$4D_2$-$4F$	5133.11	7	19481.3		pD_3	64247	
pD_3-$4F$	2846.41	10	35132	170			
pD_2-$4F$	2832.67	8	35302		$4D_2$	48594	3.0057
					$4D_3$	48418	3.0109
$4P_2$-$5S$	1649.27	12	60633	1769			
$4P_1$-$5S$	1602.52	10	62402				
					$5D_2$	29340	3.8678
$4P_2$-pD_2	1581.14	1	63246	167	$5D_3$	29298	3.8707
$4P_2$-pD_3	1576.96	4	63413	1768			
$4P_1$-pD_2	1538.14	2	65014				
					$4F$	29113	3.8829
$4P_2$-$4D_2$	1264.72	8	79068	174			
$4P_2$-$4D_3$	1261.95	15	79242	1763			
$4P_1$-$4D_2$	1237.14	15	80831		$a\bar{P}_1$	38416	
					$a\bar{P}_2$	37308	
$4P_2$-$a\bar{P}_1$	1120.52	3	89244				
$4P_2$-$a\bar{P}_2$	1106.78	8	90352	1768			
$4P_1$-$a\bar{P}_1$	1098.76	7	91012	1767			
$4P_1$-$a\bar{P}_2$	1085.55	7	92119				
$4P_2$-$5D_2$	1017.12	2	98317	41			
$4P_2$-$5D_3$	1016.69	8	98358	1769			
$4P_1$-$5D_2$	999.16	5	100084				
$4P_2$-$6S$	1075.14	3	93011	1769			
$4P_1$-$6S$	1055.08	1	94780				

II. *The Spectrum of Doubly Ionized Germanium.*—The ordinary spectrum of Ge III should consist of triple and single terms similar to Si III and Sn III. Since the author's paper[7] was written many more terms have been found, especially for the triplet system and some intercombina-

TABLE 2

SERIES SPECTRUM OF Ge III

DESIGNATION	(I. A. VAC.)	INT.			TERM	VALUES	R. D.
5^1P-5^3D_3	5623.34	6	17883				
					4^1S	259068	1.9525
4 D_1-5 P_0	5258.00	2	18019.6				
4 D_2-5 P_1	5230.90	6	19117.1	169	4^1P	167195	2.4305
4 D_1-5 P_1	5211.88	2	19186.9	459			
4 D_3-5 P_2	5136.16	10	19469.8		5^1P	80700	3.4983
4 D_2-5 P_2	5108.26	2	19576.1				
4 D_1-5 P_2	5089.21*	0	19649.4				
					5^3S	117546	2.8986
5 S-5 P_0	4292.84	10	23294.6	169			
5 S-5 P_1	4261.97	20	23463.3	459	6^3S	64975	3.8987
5 S-5 P_2	4180.21	30	23922				
5 P_2-6 S	3490.21	10	28651	459	4^3P_0	214371	2.1464
5 P_1-6 S	3435.23	7	29110	169	4^3P_1	213606	2.1503
5 P_0-6 S	3414.27*	4	29279		4^3P_2	211964	2.1586
5 P_2-5 D_1	.....	..	(30635)				
5 P_2-5 D_2	3261.16	1	30664		5^3P_0	94253	3.2333
5 P_2-5 D_3	3256.36	4	30709		5^3P_1	94084	3.2399
5 P_1-5 D_1	3216.08	1	31094	459	5^3P_2	93625	3.2479
5 P_1-5 D_2	3213.06	3	31123	169			
5 P_0-5 D_1	3198.69	1	31263				
					4^3D_1	113275	2.9528
4^3D-5^1P	3086.95	3	32395		4^3D_2	113203	2.9537
					4^3D_3	113096	2.9551
5^3S-5^1P	2714.14	4	36844 ?				
4^1S-4^3P	2199.63	20	45462				
					5^3D_1	62990	3.9596
4 D_3-4 F_2	.....	..	(47427)		5^3D_2	62961	3.9605
4 D_3-4 F_3	2107.80	0	47443		5^3D_3	62916	3.9621
4 D_3-4 F_4	2105.07	10	47501	106			
4 D_2-4 F_2	2103.75	1	47534	69			
4 D_2-4 F_3	2103.11	6	47549				
4 D_1-4 F_2	2100.70	5	47603		4^3F_2	65672	3.8780
					4^3F_3	65653	3.8785
4 P_2-$a\,\overline{P}_1$	1183.34	10	84506		4^3F_4	65595	3.8802
4 P_1-$a\,\overline{P}_0$	1173.78	9	85195	1642			
4 P_1-$a\,\overline{P}_1$	1160.79	8	86148				
4 P_2-$a\,\overline{P}_2$	1159.62	8	86235	767			
4 P_0-$a\,\overline{P}_1$	1150.55	9	86915	1644			
4 P_1-$a\,\overline{P}_2$	1137.92	10	87879				
4^1S-4^1P	1088.46	20	91873				
4 P_2-5 S	1059.03	10	94426				
4 P_1-5 S	1040.95	9	96066	1640			
4 P_0-5 S	1032.75	7	96829	763			

TABLE 2 (*Concluded*)

SERIES SPECTRUM OF Ge III

DESIGNATION	(I. A. VAC.)	INT.		TERM VALUES	R. D.
$4\,P_2$-$4\,D_1$	1013.23	1	98694		
$4\,P_2$-$4\,D_2$	1012.50	4	98765	1643	
$4\,P_2$-$4\,D_3$	1011.49	6	98873	1644	
$4\,P_1$-$4\,D_1$	996.64	4	100337		
$4\,P_1$-$4\,D_2$	995.93	5	100409	766	
$4\,P_0$-$4\,D_1$	989.09	4	101103		
$4\,P_2$-$6\,S$	680.28	2	146998		
$4\,P_1$-$6\,S$	672.76	1	148640	1642	
$4\,P_0$-$6\,S$	669.28	0	149414	774	
$4\,P_2$-$5\,D_1$	.....	..	(148978)		
$4\,P_2$-$5\,D_2$	671.05	00	149020		
$4\,P_2$-$5\,D_3$	670.88	3	149058	1635	
$4\,P_1$-$5\,D_1$	.....	..	(150619)		
$4\,P_1$-$5\,D_2$	663.77	2	150655		
$4\,P_0$-$5\,D_1$	660.52	1	151396		

tions between the two systems. The multiplets and term values with Rydberg denominators are shown in table 2. In a few cases the very faintest of the satellites do not appear.

The triplet terms are derived from 4^3P_1 the value of which is calculated from the two lines 1059.03 Å and 680.28 Å. The term values will be of only fair accuracy due to the location of these lines, but in nearly all cases they fall between the values for Si III and Sn III.

The singlet terms are found from the value of the 4^3P_1 term and the assumed resonance line 2199.63 Å. This line is very strong in the spark in air with self inductance but weaker in the vacuum spark. There may be some doubt regarding it but in the absence of all tests by the Zeeman effect it seems to be the best choice of the two or three possible lines. The other intercombination lines do not give any aid because, while they fix the value of 5^1P perhaps, the combination $4S$-5^1P falls so far in the extreme ultra-violet it could not be identified with certainty. If the resonance line is properly chosen the 4^1S term gives an ionization potential for Ge III of approximately 32 volts as compared with 16 volts for Ge II and 45 volts for Ge IV.[7]

In conclusion the author wishes to acknowledge a grant from the Research Council of Canada which enabled him to carry on this work.

[1] Lunt, *Roy. Astro. Soc.*, **85**, No. 1, 1924.
[2] Fowler, *Report on Series in Line Spectra.*
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[6] Green and Loring, *Phys. Rev.*, **30**, 574, 1927.
[7] Lang, *Ibid.*, Dec., 1927.

THE STRUCTURE AND DIMENSIONS OF THE ETHANE MOLECULE

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In a previous paper,[1] it was shown in detail that the crystal lattices of diamond and graphite could be built up of cubes having one corner shared in common between adjacent cubes. These structures showed that simple geometrical relations connected the lattice constants and the radii (half the body diagonal) of the cubes. In diamond, this radius computed from the X-ray data was found to be 0.77 Å and in graphite 0.75 Å. These results were interpreted to indicate (1) that carbon atoms in combination may be conveniently treated as cubes, (2) that the radius of the carbon atom over a wide range of conditions is approximately three-quarters of an angstrom and (3) that a union between atoms, consisting of a single electron, is a possible type of atomic union.

This paper is an account of the application of these ideas to the problem of the structure of the ethane molecule. A model of this molecule built up of cubes is first presented by giving the coördinates of the cube centers (carbon nuclei), the cube corners (electron positions) and the possible hydrogen positions referred to three axes of reference in terms of a parameter R (radius of the carbon atom). This is followed by a description of the construction of a three-dimensional scale model of the ethane molecule and after this there is a brief discussion of the properties of this model.

A Model of the Ethane Molecule.—As is well known, ethane is a saturated hydrocarbon which contains eight valency electrons due to carbon and six due to hydrogen. *Since it is saturated all the "possible electron positions" must be occupied by electrons.* This can easily be accomplished in a cubic model by uniting the two carbon atoms by a double electron bond (two cubes having an edge in common). The union between the two carbon atoms being taken care of the next question to decide concerns the nature of the bond between the carbon and hydrogen atoms. It will be recalled that both diamond and graphite have adjacent atoms united by a single electron bond (cube corner in common). Since this is the simplest type of atomic union and since there is no definite evidence in favor of any other type of bond between the carbon and hydrogen atoms in ethane this union will be assumed in this model as a first approximation. In diamond the "electron position" uniting two adjacent atoms lies on the straight line connecting the nuclei. In graphite this is not the case. Since the former bond is simpler from a geometrical standpoint it will be assumed for this model of ethane and, consequently, the radius of the carbon

atom (half the body diagonal of the cube) will be taken as 0.77 Å. And as a final approximation, it will be assumed that the "electron position" joining the hydrogen and carbon atoms is midway between the two nuclei.

It should be noted that these assumptions are arbitrary. Their purpose is to enable us to set up a definite scale model of ethane which can be quantitatively tested by comparison with experiment. One of the most rigorous tests that such a model can undergo is to build it into a lattice structure and compare the diffraction effects computed from the model with those actually observed by means of X-ray analysis. As will be shown in detail in another paper, the model herewith presented can be so arranged in a hexagonal close-packed lattice that it accounts quantitatively for all the observed diffraction effects, not only in regard to their position but also as to whether they are present or absent in the powder photographs. In this paper a description of this model is all that is intended.

Taking three rectangular axes through the nucleus of one of the carbon atoms the coördinates of all parts of this model can be expressed in terms

COÖRDINATES OF THE ETHANE MOLECULE IN TERMS OF R, THE RADIUS OF THE CARBON ATOM

Cube Centers.	(Carbon Nuclei)	
0.000,	0.000,	0.000
−0.817,	+0.471,	+1.333
Possible Hydrogen Positions		
+1.633,	±0.943,	±0.667
−1.633,	±0.943,	±0.667
0.000,	±1.886,	∓0.667
±0.817,	±1.414,	+2.000
−0.817,	+2.357,	+0.667
+0.817,	−0.471,	+0.667
−2.450,	+1.414,	+2.000
−2.450,	−0.471,	+0.667
Cube Corners.	(Electron Positions)	
±0.817,	−0.471,	−0.333
±0.817,	+0.471,	+0.333
0.000,	±0.943,	∓0.333
0.000,	0.000,	±1.000
−0.817,	+0.471,	+2.333
−1.633,	0.000,	+1.000
−1.633,	+0.943,	+1.667
−0.817,	+1.414,	+1.000
−0.817,	−0.471,	+1.667
−0.817,	−0.471,	+0.333

of a parameter R, that is to say, the radius of the carbon atom which should be equal from the considerations mentioned above to 0.77×10^{-8} cm.

The Construction of a Scale Model of Ethane.—Figure 1 shows a scale

plan of this model of ethane. The projections of the electron positions are shown by small black circles, the carbon nuclei by large cross-hatched circles, the hydrogen positions occupied by large black circles and the possible hydrogen positions unoccupied by large white circles. For a model at a scale of 5.0 cm. to the angstrom, the radii of the two circles shown in the figure are 3.6 cm. and 7.3 cm., respectively. The vertical

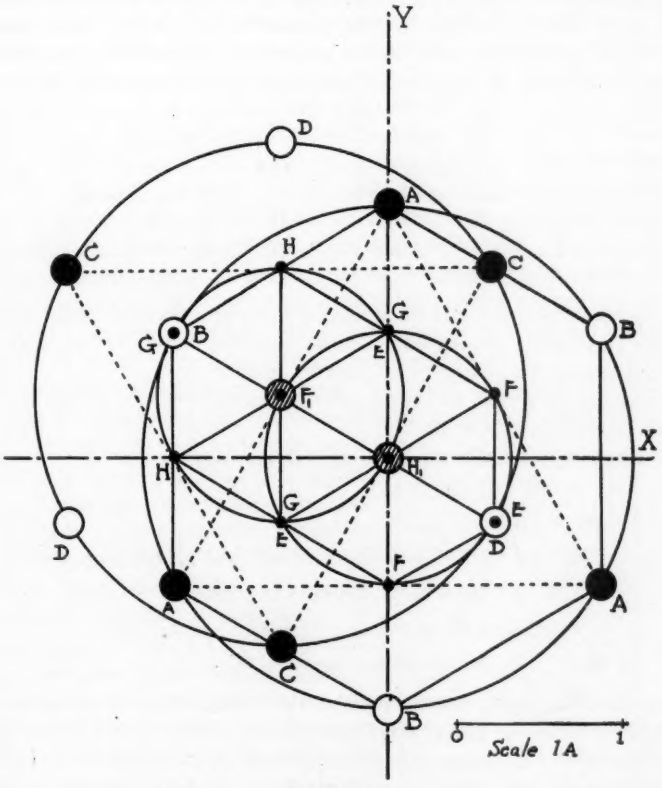
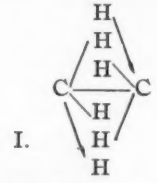


FIGURE 1

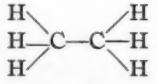
positions measured above (+) and below (−) the equatorial plane have the following values. Cube centers (carbon nuclei) H_1 −2.6 cm., (F_1) +2.6 cm., hydrogen positions occupied (A) −5.1 cm., (C) +5.1 cm., hydrogen positions unoccupied (B) 0.0 cm., (D) 0.0 cm., cube corners (electron positions) (E) −3.85 cm., (F) −1.3 cm., (H) +1.3 cm., (G) +3.85 cm., (H_1) −6.4 cm. and +1.3 cm., (F_1) +6.4 cm. and −1.3 cm.

Discussion.—The distance between the carbon atoms of this model is easily found from the coördinates given above to be 1.26 Å.

A study of the model indicates that there are twelve possible hydrogen positions and that only six hydrogen nuclei are available to fill them. This raises the question as to what six positions the hydrogen nuclei are to be assigned. A study of the coördinates indicates that six of the hydrogen positions lie in a plane. The remaining six are divided into two sets of three each and are located on the corners of two equilateral triangles lying in two parallel planes. Hence, at least two isomers are theoretically possible which can be expressed symbolically by the formulæ I and II.



I.



II.

The chemical data seems to be in accord with the second of these formulæ and for this reason the hydrogen nuclei are distributed in this model in two equilateral triangles in two parallel planes.

In conclusion, I wish to express my thanks to Professor Henry G. Gale for his interest in this work.

[1] J. K. Morse, *Proc. Nat. Acad. Sci.*, **13**, 227–232, 1927.

THE LATTICE STRUCTURE OF ETHANE

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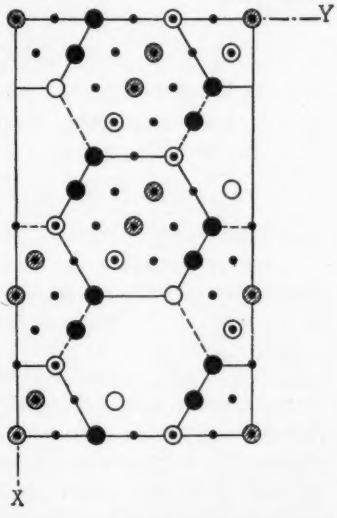
In a preceding paper[1] a scale model of the ethane molecule was presented which was based on three simplifying assumptions. (1) The bond between the carbon atoms consists of two electrons held in common. (2) The hydrogen atoms are united to their respective carbon atoms by a *single electron bond* similar to the type of union between adjacent carbon atoms in diamond[2] so that the electron position lies on the straight line joining the carbon and hydrogen nuclei. (3) This electron position lies midway between the hydrogen and carbon nuclei. As was pointed out in the paper referred to, these assumptions, especially the last two, are somewhat arbitrary and their purpose is not only to simplify but also to make definite and concrete a model which can be quantitatively established or rejected by experiment.

This paper is an account of the comparison of the X-ray diffraction effects which can be computed from a lattice built up of these ethane molecular models with the actual diffraction effects observed by Mark and Pohland[3] in their X-ray examination of solid ethane. The simple geometrical relations connecting the molecular structure with the lattice constants is first presented and the values of the lattice constants computed from them in absolute units. The coördinates of all the elements of the ethane lattice are then given in terms of the lattice constants. This is followed by a table in which the plane spacings of all planes which should give diffraction bands are compared with the plane spacings computed from the X-ray photographs. The experimental error in Mark and Pohland's experiments was estimated as three per cent, and the differences between the computed and observed values is in all cases less than this amount. Hence, not only has the molecular model passed a rigorous quantitative test and is, therefore, established but the lattice structure herewith presented is also shown to be the correct one. The importance of this result can hardly be over-estimated. If such results can be accomplished for ethane they are possible for other organic compounds and the possibility of an inductive science of chemistry or more properly chemical physics is shown to be not a dream but a reality.

The Geometrical Relations between the Molecular Structure of Ethane and Its Space Lattice.—In the ethane molecular model previously described,[1] the projections of the hydrogen atom positions belonging to the lower carbon atom lie on the corners of a regular hexagon in a plane passed through the three hydrogen atoms. In the description of the ethane model this regular hexagon is indicated in the figure by a heavy line. Figure 1 shows a plan of these hexagons assembled to form a "close packed" hexagonal lattice referred to orthorhombic axes for purposes of convenience. The hexagons lying in one horizontal plane are shown by heavy lines and those lying in the mid-plane of the lattice are indicated by a broken line. It will be noticed that each hexagon is surrounded by an empty space equal in area to six of the hexagons. So much for the arrangement of the molecules in the XY planes. Along the Z axis the ethane molecules may be considered as bounded by two parallel planes, one passed



through the three hydrogen atoms of the lower carbon atom and the other through the three hydrogen atoms of the upper carbon atom. It will be recalled that in plan a vacant space equal in extent to the space occupied by the projection of the molecule separates adjacent molecules only since these

THE COÖRDINATES OF THE ELEMENTS OF THE ETHANE LATTICE IN TERMS OF THE LATTICE CONSTANTS a_0, b_0 AND c_0

Cube Centers (Carbon Nuclei)

0 0 0	1/12, 7/12, 5/8	1/6 , 1/2 , 1/2
5/12, 7/12, 1/8	1/2, 1/2, 0	7/12, 1/12, 5/8
2/3 , 0 1/2	11/12, 1/12, 1/8	

Hydrogen Nuclei

0 1/3 , 15/16	1/12, 1/4 , 3/16	1/6, 5/6, 15/16
1/4 , 3/4 , 3/16	1/3 , 1/3 , 15/16	5/12, 1/4, 3/16
1/2 , 5/6 , 15/16,	7/12, 3/4 , 3/16	2/3, 1/3, 15/16
3/4 , 1/4 , 3/16	5/6 , 1/6 , 15/16	11/12, 3/4, 3/16
0 1/3 , 7/16	1/12, 1/4 , 11/16	1/6, 5/6, 7/16
1/4 , 3/4 , 11/16	1/3 , 1/3 , 7/16	5/12, 1/4, 11/16
1/2 , 5/6 , 7/16	7/12, 3/4 , 11/16	2/3, 1/3, 7/16
3/4 , 1/4 , 11/16	5/6 , 1/6 , 7/16	11/12, 3/4, 11/16

Hydrogen Positions Unoccupied

0 2/3, 1/16	1/12, 11/12, 1/16	1/6, 1/6, 1/16
1/4, 5/12, 1/16	1/3, 2/3, 1/16	5/12, 11/12, 1/16
1/2, 1/6, 1/16	7/12, 5/12, 1/16	2/3, 2/3, 1/16
3/4, 11/12, 1/16	5/6, 1/6, 1/16	11/12, 5/12, 1/16
0 2/3, 9/16	1/12, 11/12, 9/16	1/6, 1/6, 9/16
1/4, 5/12, 9/16	1/3, 2/3, 9/16	5/12, 11/12, 9/16
1/2, 1/6, 9/16	7/12, 5/12, 9/16	2/3, 2/3, 9/16
3/4, 11/12, 9/16	5/6, 1/6, 9/16	11/12, 5/12, 9/16

Cube Corners (Electron Positions)

0 0 3/32	0 1/6, 31/32	0 5/6, 1/32
1/12, 1/12, 1/32	1/12, 11/12, 31/32	1/3, 1/2, 3/32
1/3, 2/3, 5/32	5/12, 5/12, 5/32	5/12, 7/12, 1/32
5/12, 3/4, 3/32	1/2, 1/3, 1/32	1/2, 1/2, 3/32
1/2, 2/3, 31/32	7/12, 5/12, 31/32	7/12, 7/12, 1/32
5/6, 0 3/32	5/6, 1/6, 5/32	11/12, 1/12, 1/32
11/12, 1/4, 3/32	11/12, 11/12, 31/32	0 0 29/32
5/12, 7/12, 7/32	1/2, 1/2, 29/32	11/12, 1/12, 7/32
0 1/6, 5/32	5/12, 5/12, 31/32	1/2, 2/3, 5/32
11/12, 11/12, 5/32	0 1/2, 19/32	0 2/3, 21/32
1/12, 5/12, 21/32	1/12, 7/12, 17/32	1/12, 3/4, 19/32
1/6, 1/3, 17/32	1/6, 1/2, 13/32	1/6, 2/3, 15/32
1/4, 5/12, 15/32	1/4, 7/12, 17/32	1/2, 0 19/32
1/2, 1/6, 21/32	7/12, 1/12, 17/32	7/12, 3/12, 19/32
7/12, 11/12, 15/32	2/3, 0 13/32	2/3, 1/6, 21/32
2/3, 5/6, 17/32	3/4, 1/12, 17/32	3/4, 11/12, 15/32
1/12, 7/12, 23/32	1/12, 5/12, 15/32	1/6, 1/2, 19/32
1/6, 2/3, 21/32	7/12, 1/12, 23/32	7/12, 11/12, 21/32
2/3, 0 19/32	2/3, 1/6, 15/32	

areas are hexagonal in form they are staggered in order that they may be close packed. It seems reasonable, therefore, that a similar kind of packing should occur along the Z axis. If this is true, the distance between these planes measured along the Z axis should be exactly one-fourth of c_0. This very simple and beautiful arrangement can be expressed in the following three equations in which a_0, b_0 and c_0 are the lattice constants along the X, Y and Z axes, respectively, and R is the radius of the carbon atom (0.77 A).

$$a_0 = b_0\sqrt{3} = 4R\sqrt{6} = 7.54 \text{ Å}, \quad (1)$$
$$b_0 = 2R\sqrt{2} = 4.36 \text{ Å}, \quad (2)$$
$$c_0 = 32R/3 = 8.21 \text{ Å}. \quad (3)$$

The Diffraction Pattern of the Ethane Lattice.—Since the electron positions about each carbon atom have a very high degree of symmetry and since the mass of the hydrogen nuclei are great in comparison with the mass of the electrons, we should expect as a first approximation that the amount of X-rays scattered by the hydrogen nuclei should be negligible in comparison with the scattering due to the electrons. We can, therefore, consider that the diffraction effects produced by all the elements of the lattice are equivalent to the diffraction effects produced by scattering centers located at the positions occupied by the carbon nuclei. The computations herewith presented have been computed on the basis of this simplifying assumption. For purposes of comparison between observed and computed diffraction effects, the method proposed by Hull[4] is as striking and convenient as any which can be used for this purpose. In the following table, the last column gives the indices of the planes from which diffraction effects might be observed. Those marked absent are planes in which the lattice arrangement is such that total interference is set up and the plane should be absent from the observations. In the next column to the left is the computed plane spacing for all planes which should appear and next to it is the observed plane spacing computed from Mark and Pohland's data.

The agreement between the calculated and observed values is remarkable. Every plane which should be absent is absent in the observations and every plane which should appear is present and its plane spacing is accounted for within the experimental error with one exception—that of the 004 plane, which is marked with an asterisk in the table. When the structure factor is computed for this plane using the coördinates of the carbon atoms alone, it is seen that total interference occurs and this plane should be absent. When, on the other hand, the structure factor is computed for this plane using the coördinates of the "electron positions," it should be present but quite weak in intensity. Mark and Pohland, however, obtained a medium strong band from this plane. The abnormal intensity of this

THE CALCULATED AND OBSERVED PLANE SPACINGS OF ETHANE

Orthorhombic Axes. $a_0 = 7.54$ Å. $b_0 = 4.36$ Å. $c_0 = 8.21$ Å. Zinc X-Ray Tube Wave-Length 1.44 Å. Experimental Error Three Per Cent

INTENSITY OBSERVED	ANGLE OBS.	LOG 1 G. SIN $\theta \times 10$	D $\times 10^8$ OBSERVED	D $\times 10^8$ CALCULATED	INDICES OF PLANES
None			Absent	Absent	100
None			Absent	Absent	010
None			Absent	Absent	001
Weak	10° 6′	0.24395	4.11	4.105	002
Medium				3.77	200
Strong	10°48′	0.27273	3.84	3.78	110
None			Absent	Absent	101
None			Absent	Absent	011
				3.43	111
Strong	12°00′	0.31788	3.46	3.43	201
None			Absent	Absent	210
None			Absent	Absent	102
None			Absent	Absent	120
None			Absent	Absent	021
None			Absent	Absent	012
				2.78	202
Weak	14°42′	0.40443	2.84	2.78	112
				2.22	113
				2.22	203
				2.18	020
Very strong	19°18′	0.51919	2.18	2.18	310
None			Absent	Absent	211
None			Absent	Absent	121
Medium strong	20°42′	0.54836	2.04	2.05	004*
				1.93	022
Weak	21°30′	0.56408	1.97	1.92	312
				1.89	220
Weak	22° 6′	0.57545	1.91	1.89	400
				1.84	221
Weak	22°42′	0.58648	1.87	1.84	401
				1.80	114
Weak	23°30′	0.60070	1.81	1.80	204
None			Absent	Absent	300
None			Absent	Absent	030
None			Absent	Absent	003
None			Absent	Absent	301
None			Absent	Absent	103
None			Absent	Absent	031
None			Absent	Absent	013
				1.72	222
Weak	24°30′	0.61773	1.74	1.72	402
None			Absent	Absent	311
Very weak	27°30′	0.66441	1.56	1.56	403
Weak	28°18′	0.67586	1.52	1.50	024
Very weak	30°13′	0.70180	1.43	1.43	130
				1.41	131
Very weak	33°30′	0.74189	1.31	1.35	132

plane is of great interest and if a sufficient number of cases can be found of a similar nature they may lead to a fundamental revision of our present simple notions concerning the scattering of atoms.

In conclusion I wish to take this opportunity to express my thanks to Professor Henry G. Gale for his interest in this work.

[1] J. K. Morse, *Proc. Nat. Acad. Sci.*, **14**, 37, 1928.
[2] J. K. Morse, *Ibid.*, **13**, 227–232, 1927.
[3] H. Mark and E. Pohland, *Zeit. Krist.*, **62**, 103–109, 1925.
[4] A. W. Hull, *Phys. Rev.*, **10**, 661, 1917.

RELATIONS OF FIELD-CURRENTS TO THERMIONIC-CURRENTS

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In 1925 Millikan and Eyring[1] first developed experimentally the quantitative laws governing the extraction of electrons from metals by fields alone, i.e., the laws of "field-currents" as they called them. They first proved that the electrons constituting these field-currents are not identical with thermions as had theretofore been assumed.[2] Their most significant conclusions are stated by them in the following words, which summarize, as well as we can now do, the relations of field-currents to thermionic-currents, save for an important addition that is here made:

"(*e*) *New proof that the energy of conduction electrons is independent of temperature.* The only preceding theoretical treatment of field currents assumes them to be composed of thermions escaping under their own energy of agitation through a boundary weakened by the external field. This conception is, we think, altogether irreconcilable with the foregoing data. Indeed, the entire independence upon temperature of the field currents over a range of 700°C. constitutes new and striking evidence that 'equipartition' does not hold at all for the bulk of the conduction electrons in tungsten at ordinary temperatures. For when these conduction electrons escape as thermions, the law governing their escape is

$$i = AT^n e^{-b/T} \qquad (1)$$

in which b is the work function of Richardson. The correctness of this thermionic equation is attested by a vast amount of experimental work. Now the effect of an externally applied field of such direction as to pull out electrons should be simply to weaken the effective value of the work function b and leave i in the foregoing equation varying as rapidly with T as it always does in thermionic experiments. *The fact, then, that in*

the present experiments i is not at all dependent upon T over a 700° interval means that the electrons pulled out by the fields here used are not thermions at all. They might become such at high enough temperatures, but at temperatures up to 1000°K. the electrons here pulled out can have no assistance at all from temperature in getting out, i.e., they do not share in the thermal energy of agitation of the atoms. * * *

"The evidence (on this point) derived from the specific heats of metals is inconclusive, because the number of conduction electrons within a metal is entirely unknown. The present evidence, on the other hand, appears to be direct, accurate, and unambiguous for the non-participation of the great bulk of 'field-current electrons' (conduction electrons) in thermal motion. There must, of course, be sufficient participation by a few conduction electrons to account for the Peltier and thermo-electric effects, but this is quite consistent with the quantum theory considerations here employed. * * * *From the point of view here taken the conduction electrons are simply the lightest of the elements which, at ordinary temperatures, have taken on practically no kinetic energy, i.e., have zero specific heat, and which reach equipartition only at very high temperatures. Those of the conduction electrons which at any temperature have escaped from this zero-energy condition are the thermions.* If they carry the total atomic heat of the electrons and if this atomic heat follows the usual quantum law, then their number should increase, at low temperatures, with T^3. This would be consistent with Eq. (1).

"(*f*) *Relations of field currents and thermionic currents.* We have just seen that *at low temperatures* field currents bear no relation to thermionic currents and that their observed properties lead to the conclusion that the conduction electrons constituting them possess no agitational (temperature) energies at all. But as shown in Table VII at 1100°K. thermionic currents of measurable magnitude (10^{-12} amp.) begin to appear from tungsten and consequently the thermionic equation begins to show evidences of having a role to play in the character of the emission. Thus with increasing potential gradient, the value of the effective work function b is continually reduced and hence the value of the thermionic part of the current (the increase at 1100° over that at 300°) continually rises as is beautifully shown in Table VII so that it makes as large a percentage of the total current when the latter is 10^{-4} amperes (potential difference 5000 volts) as when it is 10^{-8} amperes (potential difference 2600 volts).

"Quite similarly, as shown in Table VI, strong fields begin at this temperature to bring into evidence the thermionic part of the current when weak fields are unable to do so. In other words, when the temperature is reached at which the number of thermions escaping without external field is at all comparable with the number constituting the field current at what we have called the critical potential gradient, the decrease in b due to

increasing field pushes up the number of emitted thermions (the electrons coming out in accordance with Eq. (1)) slightly faster than the increase in field pulls out field-electrons. Another way of saying this is that field currents are a less rapidly rising function of the field than are thermionic currents (those following Eq. (1) with b regarded as a function of the applied field F)."

In order to subject to a still further test the relations of field currents to field F, one of us (Lauritsen) in repeating in collaboration with the other the foregoing series of experiments tried the plan of plotting the logs of

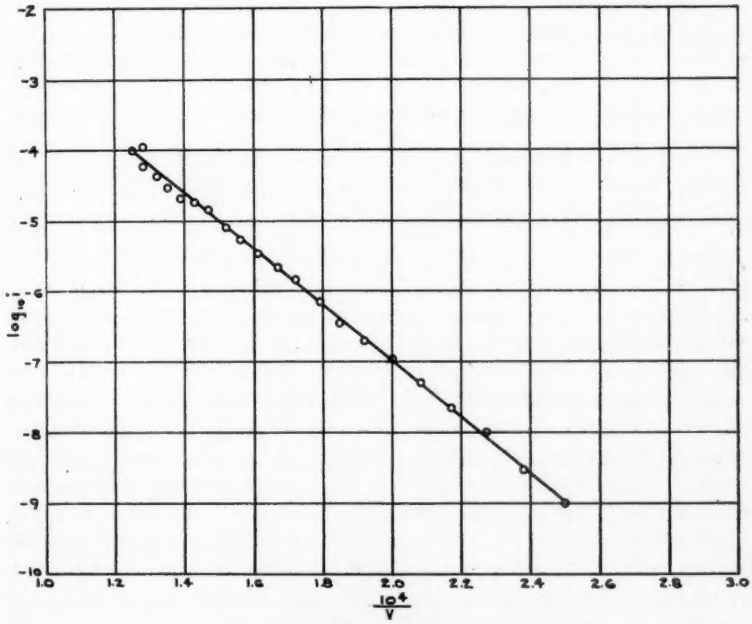

FIGURE 1

the field currents (log i) against $1/F$ instead of against F^* or $\sqrt{F}$, as had been done before. The result is shown in figure 1, which reveals the surprising fact that for these field currents log i plots as quite as good a straight line against $1/F$ as is obtained in the thermionic work when log i is plotted against $1/T$.

When, next, the Millikan and Eyring data were plotted in this new way they also yielded in every case a straight line as is illustrated in figure 2. In this particular case the range of field currents covered will be seen to be

* The abscissæ actually plotted were $10^4/V$, since for a given geometrical configuration these are proportional to the $1/F$.

from 10^{-11} amperes to nearly 10^{-3}, or an increase of more than fifty million fold.

We have now tested this relation upon the scores of field current-field relations which have been accumulated in this laboratory over a period of several years, some of them corresponding to currents drawn from fine wires surrounded with cylindrical anodes, some to currents passing between a point and a plane, some with one material and some with another, some with sharp points and some with dull ones, *without ever finding a reversible set of field-current readings which did not plot as an excellent straight line.*

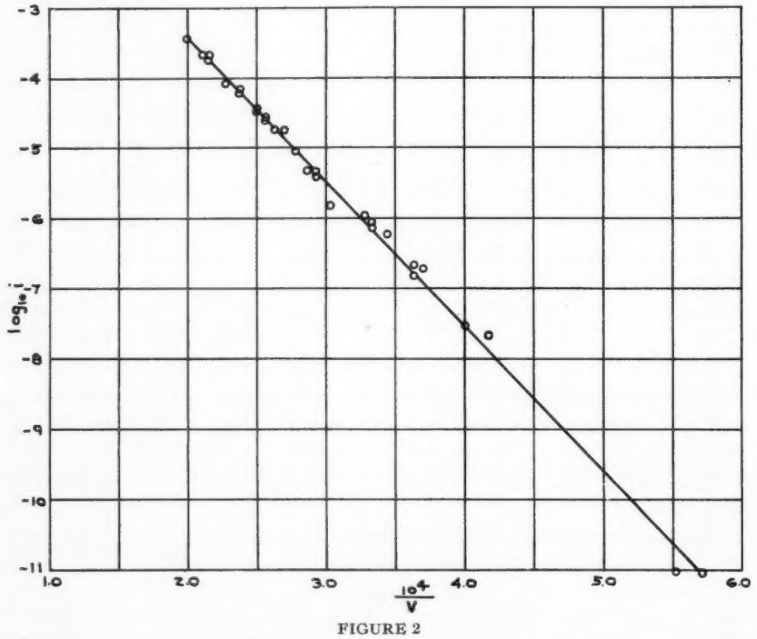

FIGURE 2

Further, in view of the fact that save at very high temperatures the field currents have been shown to be independent of temperature we are justified in considering that these $\log i - 1/F$ curves woud have precisely the form given above at absolute zero where no thermal effects whatever are possible. This statement brings sharply to the fore the conclusion that the "field-electrons" at ordinary or low temperatures have nothing to do with thermions, i.e., that they must have a static, not a dynamic, interpretation.

The facts presented in (*f*) above, however, prove definitely that at

high enough temperatures the fields do extract electrons which share in the energy of thermal agitation (thermions) as well as those which do not. *In other words, at high enough temperatures the two effects are not independent.*

We are now, therefore, in position to combine the foregoing empirical formula governing field currents, namely,

$$i = Ce^{-b/F} \tag{2}$$

with the usual formula governing thermionic currents, namely,

$$i = Ae^{-b/T} \tag{3}$$

and obtain the general formula for the extraction of electrons by fields or by temperature—or by both—in the following form

$$i = Ae^{-\frac{b}{T+cF}}. \tag{4}$$

In (2), (3) and (4) A and C may for practical purposes be taken as constants, though for the sake of more exact identification of form with the customary thermionic equation we may, if we wish, write (4) thus

$$i = A(T + cF)^2 e^{-\frac{b}{T+cF}}. \tag{5}$$

When $F = 0$ this obviously reduces to the usual thermionic equation (3) and when $T = 0$ (or is small compared to cF) it reduces to the foregoing field current equation (2) and when T is comparable with cF it yields the combination effects mentioned under (*f*). The constant b can be taken (or obtained) from thermionic data alone, and then c deduced from the slope of the straight lines shown in figures 1 and 2. The constant c, of course, changes with the condition of the surface and we have not yet taken sufficient data to know whether with a given surface one value of c will correspond to all values of T.

The new results presented in this paper may be stated thus: The application of an external field *is equivalent to increasing the temperature* of the electrons within the metal. This is nothing more than what is stated in (2) above, the T of (3) being simply replaceable by the F of (2).

We hope that it is a matter of some importance to have thus obtained a general equation which seems to embrace all the facts of thermionic-currents and field-currents combined. It may also be that the facts pictured in this equation will be of use in solving the fundamental problems of metallic conduction.

This investigation has been carried out with the aid of funds furnished by the Carnegie Corporation of New York, and administered by the Carnegie Institution of Washington.

[1] Millikan and Eyring, *Phys. Rev.*, **27**, 51, 1926.

[2] Schottky, *Zeit. Physik*, **14**, 80, 1923.

THE DENSITY, COMPRESSIBILITY AND ATOMIC WEIGHT OF NEON

BY GREGORY PAUL BAXTER AND HOWARD WARNER STARKWEATHER

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The density of neon at different pressures and at 0° has been determined by the method described in recent papers.[1]

Crude neon, containing approximately sixty per cent of helium and nitrogen, was generously supplied by the Air Reduction Company through the courtesy of Professor H. N. Davis. The gas was first subjected to chemical purification in the train shown in figure 1, by being passed successively over solid potassium hydroxide in A, hot calcium in B_1, hot copper in C_1, hot copper oxide in D_1 and D_2, potassium hydroxide solution in E, solid potassium hydroxide in A_2, phosphorus pentoxide in F, hot copper in C_2 and hot calcium in B_2 and B_3. The product, which composed less than half the volume of the original gas, was stored in the seven, 750 ml., mercury gasometers H. During the subsequent purification it was found to contain approximately 15 per cent of helium and a small proportion of nitrogen.

In common with earlier investigators[2] we found fractional adsorption to be a satisfactory method for further purification. We used chilled dehydrated chabazite as adsorbent.

Spectroscopic examination of the gas during the course of the purification under certain conditions showed the first order hydrogen lines. While it seemed probable that this was due wholly or largely to traces of moisture, to vapor from the stopcock grease, or to gases held in the tungsten electrodes of the discharge tube, the possibility remained that hydrogen had been evolved in the tubes B_2 and B_3 containing calcium. Therefore, a secondary chemical train was added to the system, through which the gas could be circulated as desired. This train consisted of hot copper oxide in D_3, phosphorus pentoxide in F_2 and hot nickel in G. Before use this train was thoroughly exhausted while the copper oxide and nickel were not. Since after the gas had been passed through this train several times the hydrogen spectrum still appeared under the same conditions as before, it is doubtful whether the neon actually contained hydrogen at any time after the chemical purification.

Owing to the limited capacity of the storage system the gas was subjected to the chemical purification in three portions. The first two portions were combined as soon as their volumes had been sufficiently reduced by physical fractionation and were further purified. Later, a third portion was brought to the same stage of purification as the first two and

added to the combination. All the final determinations were made with the mixture of all three.

For the fractional adsorption the chabazite (150 grams) was contained in the tube K, which could be connected at will with a mercury vapor pump, the gasometers H, the pump L, the spectrum tube or the density globes (shown in Fig. 3). Preparatory to each adsorption the chabazite was outgassed at 550° in a high vacuum. Ordinarily during use it was chilled with liquid nitrogen, although occasionally liquid oxygen was employed, through lack of nitrogen. When the chabazite was used to adsorb about 5 liters of pure neon at the lower temperature the residual

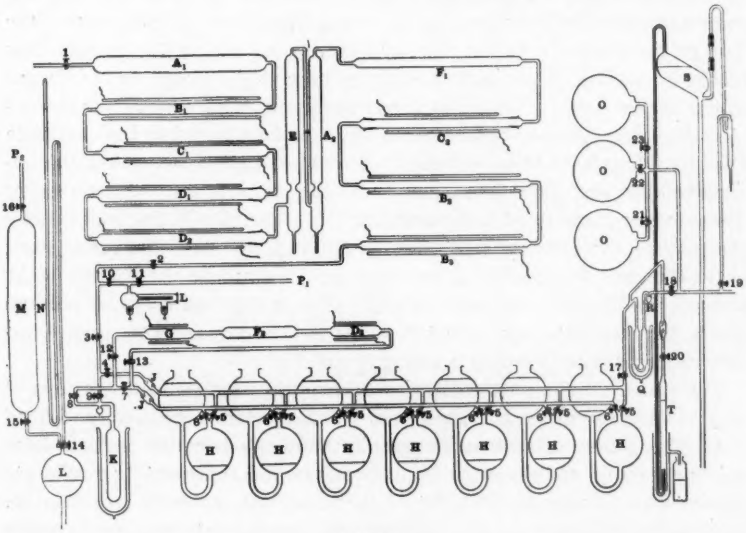

FIGURE 1

pressure of the neon was not over 15 cm., while at the higher temperature the residual pressure was 40 cm. or more. The advantage of the lower temperature is, therefore, considerable. Even at the lower temperature helium is far less adsorbed than neon, and may be rapidly though not sharply separated from neon by lowering the pressure over the chabazite. Nitrogen, on the other hand, is far more tenaciously retained by the chabazite than neon and may be eliminated by rejection of the fraction of gas remaining after the evaporation of the greater part of the neon. The proportion of nitrogen was so small that only in the first few treatments with chabazite was it evolved in detectable amounts even at room temperature. Repeated adsorption, with rejection of the least and most adsorbed fractions, was, therefore, employed for the progressive purification of the

neon. This method may be depended upon to eliminate other denser gases as well, although we found no evidence of their presence at any stage of the purification.

The progress of the purification was followed at first with a spectroscope, then by density determinations. The main portion of gas after only two adsorptions reached the stage where no impurities could be detected spectroscopically, but for several subsequent adsorptions the rejected more and less adsorbed fractions proved to contain nitrogen and helium, respectively.

The method of conducting an adsorption was as follows: The exhausted tube K containing chabazite was chilled with liquid nitrogen and put into communication with the receptacles containing neon. In the case of the storage reservoirs O, or the density globes, as soon as the pressure has reached a minimum, connection with the tube K was closed and by means of the Töpler pump S the residual gas was pumped into one of the gasometers H. Any gas in the gasometers was then transferred to the chabazite tube by filling them with mercury. After standing for some time the unabsorbed gas was allowed to expand into the pump L and then was either discarded, or transferred to the reservoir M. The middle fraction, usually the bulk of the gas, was next allowed to evaporate into the gasometers, or, in the case of a density determination, directly into the density bulbs through the chilled chabazite tube Q. The greater part of the residual gas in the chabazite was collected in the gasometers and the residue was either discarded or collected in the reservoir M.

The course of the purification by adsorption for the three portions of neon is shown in figure 2. The initial volumes refer to chemically purified gas. The course of each main sample is indicated by the vertical lines below the circle, each crossing by a horizontal line representing a separate adsorption. Circles to the right of the main line represent less volatile, those to the left more volatile fractions, the approximate volumes of which appear in the circles. If light or heavy fractions were preserved for further treatment the corresponding circles are connected by lines and arrows to others with which they were combined.

For instance, the first sample was adsorbed twice in succession, 300 ml. light fractions and 50 ml. heavy fractions being separated. The heavy fractions were rejected. The light fractions were combined and adsorbed separately. A 25 ml. middle fraction was added to the main portion. A 25 ml. heavy fraction was rejected and a 550 ml. light fraction was combined with a later one of the same size for further fractionation.

After five adsorptions the purified portions of the first two lots of neon were combined for further treatment. As soon as the mixture had been subjected to two adsorptions the first density determination was made. The result of this experiment was essentially that ultimately found as the

average of all. Four more adsorptions reduced the volume of gas so far that a third portion was brought to the same stage of purity. Owing to less liberality in the size of the light fractions rejected and to the fact that a considerable volume of helium-containing neon, resulting from the purification of the first two samples, was added to the third at an early stage, a larger number of adsorptions was necessary than with the first two

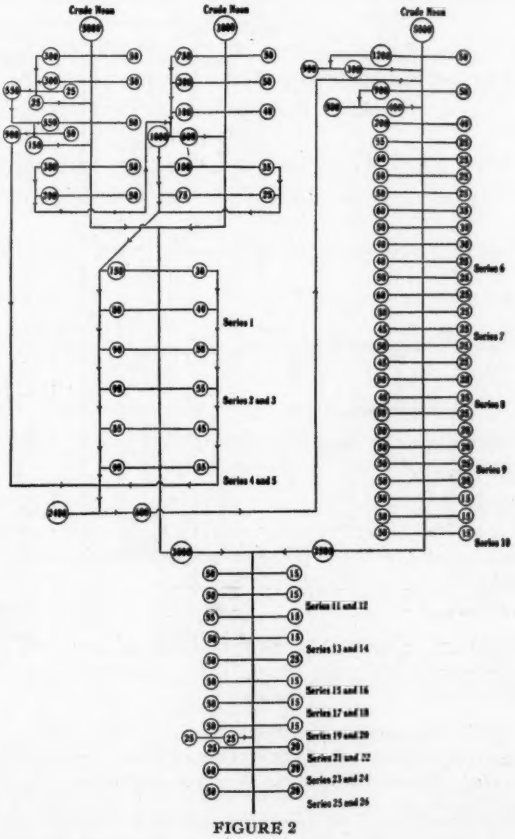

FIGURE 2

samples before the density reached the constant value already found with the earlier material. In series 6, 7 and 8 the density was gradually rising to a maximum reached in series 9 and 10. At this point all the purified material was combined for the final series (11 to 26).

The low results of series 6–8 led to the suspicion that the gas might still contain hydrogen as well as helium and caused the addition of the system

D_3, F_2, G, containing copper oxide, phosphorus pentoxide and nickel, through which all the gas was circulated several times. As stated before this produced no apparent alteration in the intensity of the hydrogen spectrum.

Aston[2] has already found that in a prolonged series of 3000 fractional adsorptions no perceptible isotopic separation of neon takes place. The

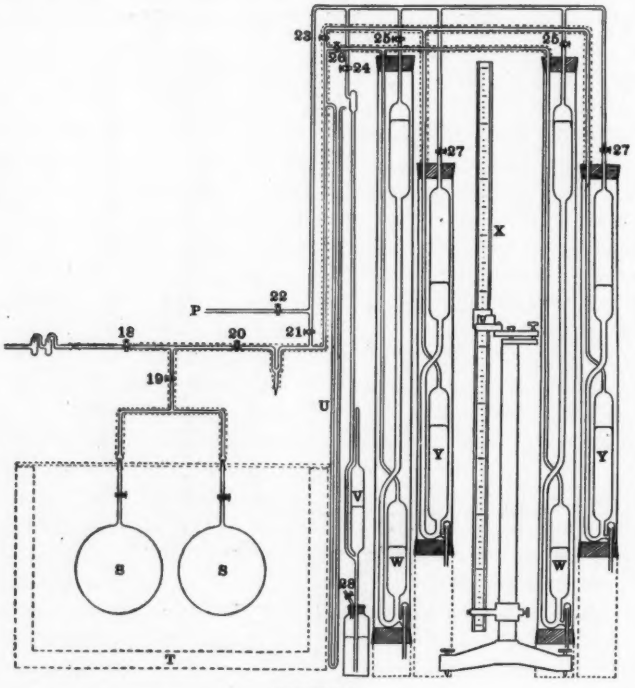

FIGURE 3

P—Lead to mercury vapor pump. *S, S*—Density Globes. *T*—Ice bath for density globes. *U*—Open arm manometer. *V*—McLeod gauge. *W, W*—High-pressure barometers. *X*—Invar meter bar. *Y, Y*—Low-pressure barometers.

essentially constant density reached by our material after a few adsorptions bears out this conclusion.

Although the apparatus for filling the globes and measuring the pressure has already been described,[3] a diagram is reproduced here.

The weights used for weighing the gas and for determining the volumes of the globes at 0° were referred by the Richards substitution method[4] to the same standard.

Corrections for the compressibilities of the globes were applied both to the weights of gas and to the volumes of the globes when filled at the lower pressure.

Pressure corrections were applied for small differences in level between the centers of the globes and the lower levels of the barometers. Since the actual pressures at which the globes were filled were always very near the reference pressures no corrections for deviations from Boyle's Law were necessary.

Since the temperature of the ice bath was usually a few thousandths of a degree below the freezing point corrections for this fact were made on the assumption that the temperature coefficient of neon is 0.003663 at 0°.

The readings of the invar meter bar were corrected for errors in the scale as determined by the Bureau of Standards, and for its coefficient of expansion.

In calculating the density of neon the following constants were employed:

Density of water at 0°	0.999868
Cubical coefficient of expansion of mercury	0.0001818
Acceleration of gravity at Lat. 45° and sea level	980.616
Acceleration of gravity at Coolidge Laboratory	980.399

No experiments have been omitted from table 1.

Earlier values found by other experimenters at one atmosphere are as follows:

Ramsay and Travers[5]	0.891
Watson[6] (determinations at 0°)	0.9000
(determinations at 16–18°)	0.9004
LeDuc[7]	0.8985
Aston[8]	0.9016

In table 2 are given the values of PV at different pressures calculated by the method of Guye[9] from the corresponding densities, on the basis of unit value for the product at one atmosphere.

If the change in PV is assumed to be proportional to the change in pressure, and equal weight is given to the values at the two lower pressures the value calculated for zero pressure is 0.99942. This value is considerably higher than that found experimentally by Burt, 0.99895.[10]

The atomic weight of neon calculated from the above values of density and deviation from Boyle's Law, as well as the corresponding values for oxygen found by us,[11] 1.42897 and 1.00092, is 20.182.

At the end of the following paper are given the results of calculating the deviation from Boyle's Law and atomic weight of neon by a more rational method.

We are very greatly indebted to the Milton Fund for Research in

Harvard University, to the BACHE AND WOLCOTT GIBBS FUNDS OF THE NATIONAL ACADEMY OF SCIENCES and to the Cyrus M. Warren Fund of

TABLE 1

THE DENSITY OF NEON

0° $g = 980.616$

SERIES	SAMPLE	NUMBER OF ADSORPTIONS	GLOBE IV 2110.95 ML.	GLOBE VII 2117.77 ML.	AVERAGE
		$P = 760$ mm.			
1	1	7		0.89991	
2	1	9		0.89988	
4	1	11		0.89987	
			Average	0.89989	
6	2	11		(0.89785)	
7	2	15		(0.89892)	
8	2	19		(0.89948)	
9	2	23		0.89992	
10	2	27		0.89990	
			Average	0.89991	
11	1 + 2	13 and 29	0.89991	0.89994	0.89993
13	1 + 2	15 and 31	0.89987	0.89996	0.89992
15	1 + 2	17 and 33	0.89988	0.89991	0.89990
17	1 + 2	18 and 34	0.89981	0.89990	0.89986
19	1 + 2	19 and 35	0.89993	0.89992	0.89993
21	1 + 2	20 and 36	0.89984	0.89997	0.89992
		Average	0.89987	0.89993	0.89991
Average of all except series 6, 7 and 8					0.89990
		$P = 506.667$ mm.			
3	1	9		0.60000	0.60000
12	1 + 2	13 and 29	0.60000	0.60007	0.60004
16	1 + 2	17 and 33	0.60007	0.60010	0.60009
18	1 + 2	18 and 34	0.60002	0.60014	0.60008
23	1 + 2	21 and 37	0.59999	0.60006	0.60003
25	1 + 2	22 and 38	0.59998	0.60006	0.60002
		Average	0.60001	0.60007	0.60004
		$P = 253.333$ mm.			
5	1	11	0.30000	0.30007	0.30004
14	1 + 2	15 and 31	0.30007	0.30009	0.30008
20	1 + 2	19 and 35	0.30003	0.30010	0.30007
22	1 + 2	20 and 36	0.30009	0.30018	0.30014
24	1 + 2	21 and 37	0.30012	0.30010	0.30011
26	1 + 2	22 and 38	0.30011	0.30012	0.30012
		Average	0.30007	0.30011	0.30009

TABLE 2

PRESSURES ATMOSPHERES	Pv
1	1.00000
2/3	0.99982
1/3	0.99959

the American Academy of Arts and Sciences for generous assistance, without which this work would not have been possible.

[1] Baxter and Starkweather, these PROCEEDINGS, **10**, 479 (1924); **11**, 231 (1925); **12**, 20, 699, 703 (1926).

[2] Watson, *J. Chem. Soc.*, **97**, 810 (1910); Aston, *Phil. Mag.*, **37**, 523 (1919).

[3] See Baxter and Starkweather, these PROCEEDINGS, **12**, 701 (1926).

[4] Richards, *J. Amer. Chem. Soc.*, **22**, 144 (1900).

[5] Ramsay and Travers, *Trans. Roy. Soc.*, **197**, 47 (1901).

[6] Watson, *J. Chem. Soc.*, **97**, 810 (1910).

[7] LeDuc, *Compt. Rend.*, **158**, 864 (1914); *Ann. Physique* [9] **9**, 24 (1918).

[8] Aston, *Proc. Roy. Soc.*, **A89**, 439 (1914); *Phil. Mag.*, **37**, 523 (1919). This result is corrected by us for deviation of neon from Boyle's law.

[9] Guye, *J. chim. phys.*, **17**, 171 (1919).

[10] Burt, *Trans. Far. Soc.*, **6**, 19 (1910).

[11] Baxter and Starkweather, these PROCEEDINGS, **12**, 699 (1926).

THE DENSITY, COMPRESSIBILITY AND ATOMIC WEIGHT OF ARGON

BY GREGORY PAUL BAXTER AND HOWARD WARNER STARKWEATHER

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In two recent discussions of earlier work upon the density of argon, by Blanchard and Pickering,[1] and Moles,[2] the more probable values 1.7825 and 1.7833, respectively, are deduced. Since both these figures involve corrections of somewhat dubious validity, the atomic weight of argon derived from them is uncertain in the second decimal place.

We have applied to argon the methods utilized in determining the densities and compressibilities of oxygen, nitrogen, helium and neon,[3] and find the normal density 1.78364 and the limiting density 1.78204. The atomic weight of argon computed from these figures is 39.943.

Argon was very generously provided by the Air Reduction Co., through the courtesy of Mr. C. C. Van Nuys. The gas as furnished was stated to be nearly 99 per cent pure, the impurity being chiefly nitrogen with a trace of oxygen. In the course of the purification which we employed we were unable to detect any of the less abundant rare gases. The chemical purification train consisted of solid potassium hydroxide in A (Fig. 1), hot copper in B, hot copper oxide in C_1 and C_2, aqueous potassium hydroxide in D, solid potassium hydroxide in E, phosphorus pentoxide in F_1, hot calcium in G_1 and G_2, hot copper oxide in C_3, phosphorus pentoxide in F_2 and hot nickel in H. The U-tube J contained chabazite chilled with ice, and the tube K contained chabazite chilled with liquid air. In L the first portion of chemically purified argon was liquefied, and the remainder of

the gas was caused to bubble through this liquid on its way to M where it also was liquefied.

Fractional distillation from M to O, O to Q and Q to S followed. In each distillation the first portion of gas was liquefied in one of the intermediate receptacles N, P and R, and the remainder was caused to bubble through the liquid argon contained in this vessel on its way to the receiver. The most and least volatile portions were rejected in each distillation. Danger from excessive pressure in the apparatus was avoided by mercury

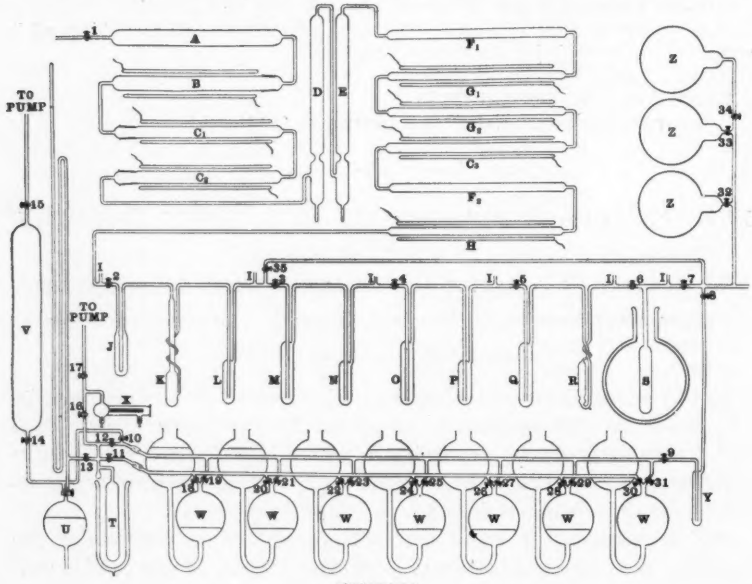

FIGURE 1

traps connected at the points I. The final product was stored in S which was chilled with liquid air.

A density determination made with the most volatile portion of the argon gave so low a result, 1.777, that it seemed probable that the gas was still contaminated with nitrogen, a conclusion easily substantiated by spectroscopic examination. Apparently the nitrogen had been very imperfectly removed by the calcium owing to too cautious heating, and also by the fractional distillation. The latter difficulty has previously been met by others.

So far as we know, fractional adsorption on chilled charcoal was first used by Valentine and Schmidt[4] for the purification of argon. We turned next to this method, but using chilled dehydrated chabazite for the purpose instead of charcoal. The chabazite was contained in the tube T and was

chilled with either liquid oxygen or ether and solid carbon dioxide. When the argon was adsorbed upon the chilled chabazite and the latter was allowed to warm up slowly, the first portion of gas when tested spectroscopically showed no evidence of nitrogen, but with gradually rising temperature the spectrum of nitrogen became more and more evident.

The procedure finally adopted was, in brief, as follows: The chabazite was chilled with liquid oxygen and was nearly saturated with argon at that temperature. Even when nearly 20 liters of argon had been taken up the residual pressure was only a few millimeters. The temperature was then raised to $-78°$ (ether and carbon dioxide). By means of the pump U a considerable fraction of gas was removed. Although no impurity whatever could be detected spectroscopically in this portion, nevertheless it was always rejected. Next, while the chabazite was still chilled with carbon dioxide, the major portion of adsorbed gas was removed by means of the pump U and stored in the gasometers H, the reservoirs Z or the density globes. Nitrogen was never detected in this fraction even in the initial adsorption. Then the chabazite was warmed to room temperature and the residual argon was gradually pumped off. In the earlier adsorptions this fraction was found to contain traces of nitrogen and was rejected, as was that evolved when the chabazite was outgassed at 550°. In the later adsorptions the gas evolved at room temperature was stored and added to the main portion in the next adsorption, although it was never used for filling the globes in a density determination. The apparatus was so constructed that communication could be established at will between the chabazite tube T, the reservoirs Z, the gasometers H and the density globes.

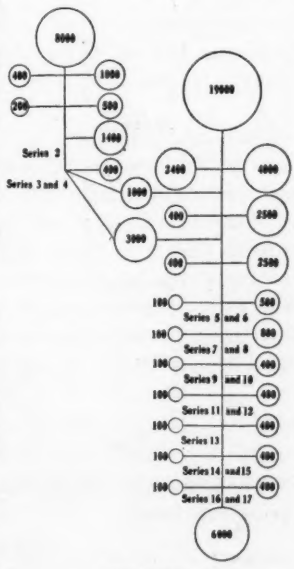

FIGURE 2

After three adsorptions density determinations were begun with the middle fraction of gas from each adsorption, which was allowed to evaporate directly into the globes from the chabazite through a chilled trap Y. No upward trend in the density results appeared with continued purification of the gas. In fact, a slight downward tendency was apparent in the determinations at one atmosphere, although not at the lower pressures.

The course of the purification by adsorption for the two portions of argon is shown in figure 2. The initial volumes refer to chemically purified gas. The course of each main sample

is indicated by the vertical lines below the circle, each crossing by a horizontal line representing a separate adsorption. Circles to the right of the main line represent less volatile, those to the left more volatile fractions, the approximate volumes of which appear in the circles. After four adsorptions the product from the first portion of argon was added to the second portion at the points indicated.

The apparatus for filling the globes and measuring the pressure was identical with that used for neon,[5] and the various constants employed and corrections applied were similar.

No experiments are omitted from the following table. Globe VII was broken after series 13 had been completed.

TABLE 1

THE DENSITY OF ARGON

0° $g = 980.616$

SERIES	NUMBER OF ADSORPTIONS	GLOBE IV 2110.95 ML.	GLOBE VII 2117.77 ML.	AVERAGE
		$P = 760$ mm.		
2	3	1.78380	1.78378	1.78379
3	4	1.78366	1.78362	1.78364
5	4	1.78363	1.78364	1.78364
7	5	1.78361	1.78356	1.78359
9	6	1.78378	1.78368	1.78373
11	7	1.78356	1.78358	1.78357
13	8	1.78353	1.78361	1.78357
14	9	1.78355		1.78355
Average		1.78364	1.78364	1.78364
		$P = 506.667$ mm.		
4	4	1.18866	1.18869	1.18868
8	5	1.18877	1.18882	1.18880
10	6	1.18878	1.18875	1.18877
16	10	1.18868		1.18868
Average		1.18872	1.18875	1.18874
		$P = 253.333$ mm.		
6	4	0.59420	0.59418	0.59419
12	7	0.59420	0.59413	0.59417
15	9	0.59422		0.59422
17	10	0.59423		0.59423
Average		0.59421	0.59416	0.59419

The individual results of earlier experiments at one atmosphere by others are given below.

Ramsay[6]	1.7817*	
Lord Rayleigh[7]	1.7808*	Corrected by Guye 1.7825 Corrected by Moles 1.7832
Ramsay and Travers[8]	1.7824	
Fisher and Ringe[9]	1.7809*	

Fisher and Hahnel[10]	1.7813*	
Schultze[11]	1.78376	Corrected by Blanchard and Pickering 1.7824
Holst and Hamburger[12]	1.7834	
Leduc[13]	1.7825	

* Contained other rare gases.

On the basis of unit value for the product PV at one atmosphere the following values for the product at lower pressures are calculated from our results for the densities at different pressures by the method of Guye.[14]

TABLE 2

PRESSURE ATMOSPHERES	PV
1	1.00000
2/3	1.00030
1/3	1.00060

Within the error of the experiments PV changes regularly with the pressure, and the extrapolated value for zero pressure is 1.00090. This value is in fair accord with the findings of others, which are given below.

Lord Rayleigh[15]	1.00093
Onnes[16]	1.00077
Holborn and Schultze[17]	1.0009
Leduc[18] at 14°*	1.00078

From the limiting value of PV and the normal density of argon the limiting density may be computed to be 1.78204, and if the corresponding values for oxygen are taken as 1.00092 and 1.42897,[19] the calculated value of the atomic weight of argon is 39.943.

The conventional method of calculating the deviation from Boyle's Law makes the questionable assumption that the value of PV for one atmosphere is correct. Although the uncertainty introduced in this way is not large, it is on the whole better to find the best straight line[20] to represent the observed values of some simple function of the density plotted against the pressure, and then to extrapolate to zero pressure. This has been done for our observations on oxygen, nitrogen, neon and argon. The average values of PV/w at different pressures were weighted inversely as their probable errors,[21] and combined by the method of least squares into the following equations:

Oxygen	$PV/w = 0.699807\ (1 + 0.000927(1-p))$
Nitrogen	$PV/w = 0.799763\ (1 + 0.000401(1-p))$
Neon	$PV/w = 1.111237\ (1 - 0.000592(1-p))$
Argon	$PV/w = 0.560651\ (1 + 0.000901(1-p))$

From these equations the following values result:

		OXYGEN	NITROGEN	NEON	ARGON
Normal Density	calc.	1.428965	1.25037	0.89990	1.78364
	obs.	1.428965	1.25036	0.89990	1.78364
Density at 3/4 atm.	calc.	1.07148			

		OXYGEN	NITROGEN	NEON	ARGON
	obs.	1.07149			
Density at 1/2 atm.	calc.	0.71415			
	obs.	0.71415			
Density at 1/4 atm.	calc.	0.35699			
	obs.	0.35699			
Density at 2/3 atm.	calc.		0.83347	0.60005	1.18874
	obs.		0.83348	0.60004	1.18874
Density at 1/3 atm.	calc.		0.41668	0.30008	0.59419
	obs.		0.41667	0.30009	0.59419
Limiting Density		1.42764	1.24987	0.90043	1.78204
Atomic Weight		16.000	14.008	20.183	39.944
$(PV)_0/(PV)_1$		1.00093	1.00040	0.99941	1.00090

In the case of oxygen the average densities obtained in series 4, 5 and 6 with one liter globes[22] and in the series with two liter globes were weighted inversely as their probable errors and combined in the weighted average given in the above table.

It is reassuring that this method of distributing errors produces such small differences between the calculated and observed values of the densities at different pressures, for in only six cases is this difference as large as 0.01 mg. The effect upon the calculated atomic weight is correspondingly small, for it is always less than 0.001 unit, although fortuitously the third decimal place is affected in the case of neon and argon.

The above value of the limiting density of oxygen yields as the limiting value of molal volume 22.4146 liters (g = 980.616).

We are very greatly indebted to Professor Harvey N. Davis for assistance in the mathematical treatment of the results; also to the Milton Fund for Research in Harvard University, to the BACHE AND WOLCOTT GIBBS FUNDS OF THE NATIONAL ACADEMY OF SCIENCES and to the Cyrus M. Warren Fund of the American Academy of Arts and Sciences for generous assistance, without which this work would not have been possible.

[1] Blanchard and Pickering, *Bur. Standards Sci. Paper* No. 529 (1925).

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[4] Valentine and Schmidt, *Ann. Physik.*, **18**, 187 (1905).

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[9] Fisher and Ringe, *Ber.*, **41**, 2017 (1908).

[10] Fisher and Hahnel, *Ibid.*, **43**, 1441 (1910).

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[12] Holst and Hamburger, *Z. phys. Chem.*, **91**, 513 (1916).

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[14] Guye, *J. Chim. Phys.*, **17**, 171 (1919).

[15] Lord Rayleigh, *Z. physik. Chem.*, **52**, 705 (1905).

[16] Onnes and Crommelin, *Comm. Phys. Lab. Leiden*, No. 118b (1910).
[17] Holborn and Schultze, *Ann. Physik*, **47**, 1089 (1915).
[18] Leduc, *Compt. rend.*, **167**, 70 (1918).
[19] Baxter and Starkweather, these PROCEEDINGS, **12**, 702 (1926).
[20] The experimental evidence is not accurate enough to determine the curvature of this line.
[21] Calculated by means of the expression $\frac{\Sigma|d|}{n\sqrt{n-1}}$.
[22] Corrected for an erroneous value of g employed in the original calculations.

THE CHROMOSOMES OF PELLIA NEESIANA

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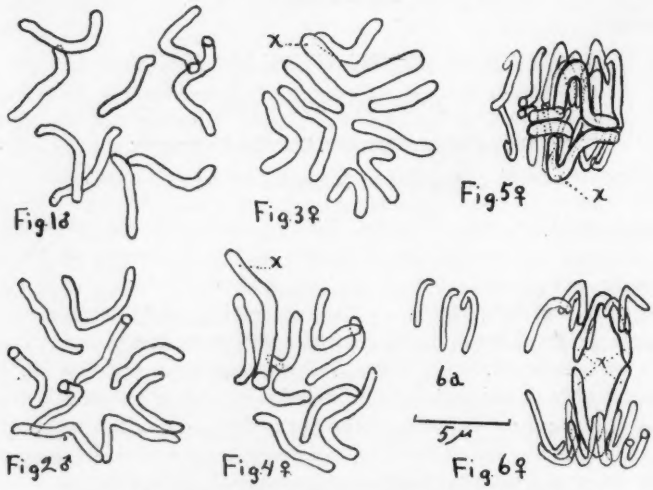
Female plants of *Pellia Neesiana* Limpr. collected in the Black Forest and cultivated at the Botanical Garden of the University of Freiburg (Germany) were inseminated (Aug., 1926) and material was fixed for cytological studies of fertilization. At the same time, and later, male plants also were fixed. This material showed evidence of the presence in this species of sex-chromosomes of the *Sphaerocarpos* type but this evidence was withheld from publication to be included in a paper on somatic mitosis in this genus and in *Riccardia*. The appearance of Lorbeer's paper[1] describing in this species and in *P. Fabbroniana* a type of sex-chromosomes totally different from that found in my material, and of the more recent paper of Heitz[2] on the chromosome numbers in these and other species, seem to call for the publication of a summary of my results.

The male plant of *P. Neesiana* has nine chromosomes. These differ somewhat in size, but no one chromosome is markedly larger than the others, while the smallest corresponds approximately to the smallest one in the female plant (Figs. 1, 2).

The female plant also has nine chromosomes, but of these one is about twice as large as any other or as any of the chromosomes of the male plant (Figs. 3, 4). The apparent analogy of this large chromosome to the sex-chromosome in the female plant of *Sphaerocarpos* seems to justify its being called the X-chromosome in accordance with current usage. It is both thicker and longer than any of its fellows and is easily distinguished from them in metaphase figures. It is even more readily recognizable in the late anaphases, during which it tends to lag slightly on the spindle (Figs. 5, 6), and in the early telophases, during which it remains a fairly smooth homogeneous rod while the substance of the other chromosomes

disperses and reconstitutes the metabolic nucleus (Figs. 7–10). Occasionally the end (or ends) of the X-chromosome seem to project out of the newly constituted daughter nucleus (Fig. 7). In this case the nuclear membrane seems to join the surface of the X-chromosome but in later stages this membrane can be traced over the projecting end of the chromosome.

In the late telophases and interphases the surface of the X-chromosome becomes irregular and a part of its substance disperses as granules of various sizes (Fig. 10). In the meristematic region of the female thallus a considerable portion of this chromosome remains condensed throughout



Figures 1 and 2.—Metaphase chromosomes of male thallus.
Figures 3 and 4.—Metaphase chromosomes of female thallus.
Figure 5.—Early anaphase in female thallus; portion of X-chromosome slightly displaced by microtome knife.
Figure 6.—Late anaphase in female thallus. One autosome and parts of two others were removed in sectioning (6*a*).

the interphases. This recalls the behavior of the sex-chromosomes in the spermatogonia of many animals but it seems to be the first known case of such behavior of sex-chromosomes in plants. It is also the first case known to me of such behavior in somatic mitosis of either plants or animals. It should be remembered, however, that I am dealing with the monoploid sexual generation. I have not yet studied mitosis in the sporophyte nor have I recognized with certainty the X-chromosome in a preliminary examination of the one diploid gametophyte.[3]

One occasionally finds in male plants late telophase and interphase nuclei which show series of large opaque granules or groups of granules,

but I am as yet unable to find that they have any constant relation to any one chromosome. Certainly no chromosome of the male is closely comparable in its telophasic behavior to the X-chromosome of the female

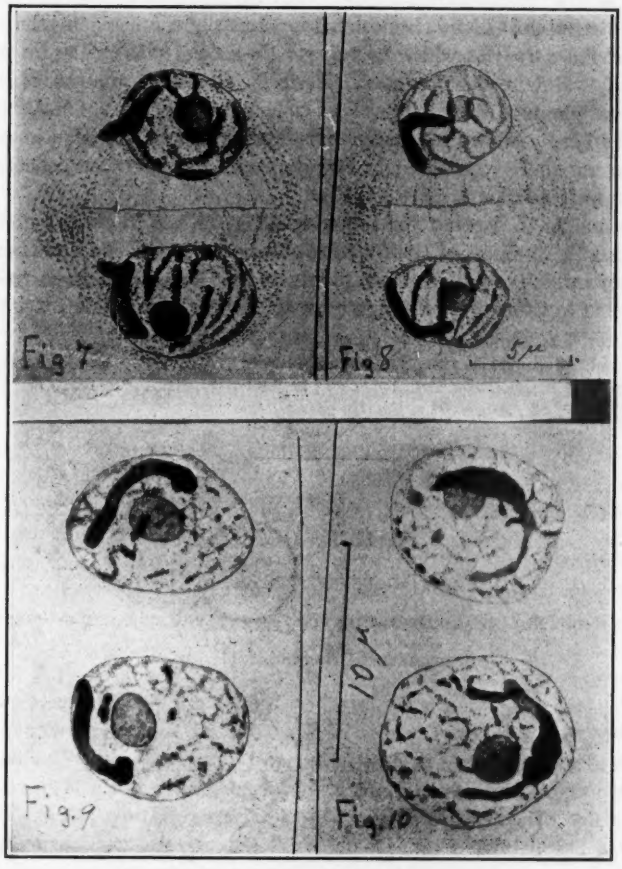


Figure 7.—Early telophase in female thallus.
Figure 8.—Medium telophase in female thallus.
Figure 9.—Late telophase in female thallus.
Figure 10.—Early interphase in female thallus.

thallus. It seems reasonable to suppose that one of the chromosomes of the male plant is the synaptic mate of the X-chromosome and should be called the Y-chromosome, but I am as yet unable to identify it in the male thallus.

Lorbeer[1] describes in the male plant seven chromosomes (autosomes), and in the female seven autosomes plus two X-chromosomes. He postulates the origin of this condition by an habitual meiotic non-disjunction of one pair in a monoicous ancestor having eight chromosomes (monoploid). In support of this hypothesis he cites the condition in the monoicous *Pellia epiphylla* which he reports (in accord with earlier investigators) to have eight chromosomes. Heitz[2] finds nine chromosomes in the monoploid plants of this latter species collected in Germany and Wolfson[4] (who has kindly shown me his preparations) finds clearly nine in plants of the same species collected in Belgium and also in those collected in America (station near Homer, N. Y.).

I am unable to account for the wide divergence of Lorbeer's observations from the findings of Heitz and Wolfson in *P. epiphylla* and from the conditions which obtain in my plants of *P. Neesiana*. It is perhaps permissible to wait for a reëxamination of Lorbeer's material.

[1] Lorbeer, G., *Zeitschr. ind. Abst. Vererb.*, **44** (1–109), 1927.
[2] Heitz, E., *Abh. Nat. Verein, Hamburg*, **21** (48–58), 1927.
[3] Showalter, A. M., *Proc. Nat. Acad. Sci.*, **13** (369–372), 1927.
[4] Wolfson, A. M., *Amer. Jour. Bot.* (in press).

THE PRODUCTION OF VARIATION IN NICOTIANA SPECIES BY X-RAY TREATMENT OF SEX CELLS

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During the past year an effort has been made to produce heritable variations in a number of species of *Nicotiana* by X-ray treatment of the sex cells at various stages in their maturity. Positive results of a character never before obtained with plant material have apparently been secured. In the case of *N. tabacum* var. *purpurea** two mature plants bearing numerous flower buds of all ages up to anthesis were X-rayed last January and two other equivalent plants in September received identical treatment.

A Coolidge X-ray tube with an air-cooled tungsten target was used, operated by a transformer without rectification, at an effective potential of 50,000 volts. No effort was made to determine the exact distance between the target and each of the numerous buds borne on the terminal inflorescences. The average distance was approximately 30 cm. The current through the tube was 2.8 milliamperes. The time of exposure was 10 minutes for the one plant and 20 minutes for the other plant in

the January X-raying, and 15 minutes for each plant in the September X-raying.

The past summer over 1000 plants from the first X-raying came to maturity in the field and greenhouse. Seven populations from seed produced in a corresponding number of capsules from X-rayed buds which had been allowed to self-pollinate, made up this total. Over 20% of variant plants appeared; in one population, which was most intensively studied, 136 out of 168 plants were "variant." If a corresponding search for off-type plants within the other six populations had been made the number of variants in the total would undoubtedly have approximated 50%. These variants, in general, showed striking alteration of some one or of all the external morphological characters which distinguish *tabacum* var. *purpurea*. In a few cases the distinction between variant and control lay largely in the reduced fertility of the former, but in most instances such reduction in fertility was accompanied by a modification of one or more characters, appreciable but sometimes not accurately measurable. The majority of the variants were at first glance seen to be decidedly abnormal. No two were identical and only a rough classification into the following types was successful. In habit the variants were dwarf, low or tall; the leaves were large, small, lanceolate, cordate, smooth, wavy, corrugated, dark green, gray-green, petiolate—the auricles being absent, smooth, ruffled, decurrent. The flowers were very small, small, long, fluted, calycine, folded, notched, ten-sided, light pink, pink, lively red, reddish purple, with 2-, 3- or 4-celled ovaries. These general characterizations, distinguishing variant characters from normal, used in classification are included because they give some picture of the extent of variation observed. The majority of the variants exhibited some reduction in fertility, but only rarely were they completely sterile, while a considerable number were completely fertile.

From the seed of the September X-raying over 500 seedlings will be in flower within a month—some from selfed seed of X-rayed buds, some from X-rayed male and untreated female. In both cases seedlings possessing the same habit and leaf variation as those which produced the variants from the Janauary X-raying are conspicuously present. In other words, it would appear that we now have a method of producing at will an extensive series of variations in *N. tabacum*, it being necessary to expose to X-rays only the male sex cells. Over 500 seedlings from selfed seed of 5 variants from the January X-raying are also growing in the greenhouse. In seedling characters there is again clear evidence of the occurrence of variants in this second generation from X-rayed sex cells.

Cytological study has been made of some 15 decidedly variant plants of the January X-raying. In nearly half of them there appear to be 24_{II} at I M just as in the control and chromosome behavior is normal,

the tetrads showing practically no microcytes. The remainder show cytological abnormalities of all kinds, both in somatic and meiotic divisions. In every case where counts can be made in P. M. C. the chromosome number is less than normal. In a few sterile plants, the external morphology of which is highly abnormal, degeneration sets in, in P. M. C., either in archesporial stages or during early prophases with the result that almost no pollen of any sort is matured. In some partially fertile variants similar degeneration of sporogenous tissue will be characteristic of one bud or a part of a bud, other buds or the remainder of the bud showing entirely normal maturation stages. In one plant only a small fraction of the normal number of ovules is formed, the placentae being bare over considerable areas.

Six variants have been given detailed cytological examination, using paraffin sections in addition to acetocarmine smears. In three of them an exceptional situation obtains. At I M and at II M one univalent partner, apparently of normal size, possesses a small appendage which is in each case of the same shape and size and is attached by a delicate thread to the parent chromosome. In appearance this appendage suggests strongly the satellite now known to be characteristic of somatic chromosomes in many genera including *Nicotiana*. Such an appendage on a meiotic chromosome has never been seen in the course of many years' study of chromosome behavior in many species and intraspecific hybrids in *Nicotiana*. Its occurrence in half the individuals intensively studied suggests that X-ray bombardment may commonly produce a change in chromosome morphology of a sort never before observed. In one plant exhibiting this chromosome appendage there are only 21 normal pairs, 1 pair, a member of which bears an appendage, and one unpaired chromosome which has been structurally modified to such an extent that, though at I M it has the characteristic appearance of a small unpaired *tabacum* chromosome, at diaphase (diakinesis) and in somatic anaphases it is much elongated and partially fragmented. We have in our cultures a number of plants of *tabacum* haploid and a race possessing only 23 pairs of chromosomes, but it is surprising to find that a fertile soma can be built with two less than a full haploid set present.

Other variants show $23_{II} + 2_{I}$, $23_{II} + 1_{I}$, $22_{II} + 2_{I}$ or 22 pairs and one pair, a member of which bears an appendage. Cases of this sort are not simply the result of an increase in the incidence of non-disjunction. In one plant a number of P. M. C. at I M and II M showed 48 in place of 24 chromosomes. Practically all of the plants studied cytologically were to some extent fertile, and some of them were quite highly so.

Sex cells of a number of other species of *Nicotiana* have been X-rayed during the past year. The same dosage as that given the buds of *tabacum* var. *purpurea* was applied in each case. Small populations from X-rayed

sex cells of *rustica* var. *pumila* and *Bigelovii* var. *Wallacei* were grown last summer. There was no evidence of variation in the latter and no appreciable reduction in fertility. In the case of *rustica* there were both dwarfs and plants of normal size, but little decrease in fertility in the former. The treatment was apparently gametically or zygotically lethal for *suaveolens* and almost so in the case of *nudicaulis*. The former produced non-viable seeds only, and from seed of the latter species only two plants grew to maturity. One of them was normal in appearance and in flower, while the other was decidedly dwarf and set no seed until late in the season and then only in cleistogamous flowers. It would seem probable that progenies from both variant and "normal" plants of the two X-rayings of *tabacum* will exhibit recessive modifications, as may progenies from the other species which received similar treatment. It is proposed as soon as possible to study the effects of variations in X-ray dosage on the nature of the heritable variations which we now know can be obtained, and possibly to determine comparative effects of X-ray treatment of seeds of species in which variation has already been produced by treatment of sex cells. A complete account of the cytology and genetics of the X-ray variants will appear in the *Botanical Gazette*.

* A tall, large-leaved, red-flowered variety of the species of commerce, grown in the pure line for many years and extensively studied here in inter- and intraspecific hybrids (cf. *Univ. Calif. Publ. Botany*, vols. 5 and 11, and elsewhere).

GENETIC EFFECTS OF X-RAYS IN MAIZE

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Crossing-Over.—The frequency of crossing-over in the *C-Sh-Wx* region in maize, though varying rather widely in different plants, is remarkably constant within the individual. Repeated pollinations from the same heterozygous plant throughout the pollen-shedding period show no significant variation in cross-over percentage, indicating (for the male germ cells) that considerable differences in age and wide fluctuation in temperature and other weather conditions have no appreciable effect on crossing-over in this region.[2] The discovery by Mavor[3] that cross-over frequency in Drosophila is affected by X-rays even in the sex chromosome (in which Plough[4] and Bridges[5] had found crossing-over unaffected by age and temperature differences) suggested that cross-over variations in the *C-Sh-Wx* region might be induced by X-ray treatment. Later experiments by Muller[6] indicated that certain regions in Drosophila are relatively immune to the effects of X-rays on crossing-over.

In 1925, 40 plants of three families heterozygous for *C-Sh-Wx* were treated by X-raying the young tassels at the time of the maturation divisions. The stage in the development of the germ cells was determined by cytological examination of several representative plants in each family, and the treatments were applied at about the middle of the maturation period. A study of the course of maturation in several untreated plants, made by daily examination of anthers in aceto-carmine smears, had shown that the reduction divisions in the main tassel occur mostly during a period of five or six days, and that they follow a regular course through the inflorescence, so that the stage of development may be fairly accurately estimated by a single examination at any time during the period. In order to increase the proportion of germ cells exposed to treatment at any susceptible stage, three of the four series of treatments were applied intermittently, in three exposures at 48-hour intervals.

The treatments were applied in the open field, by means of a General Electric Portable X-ray Outfit, with a 10 milliampere self-rectifyng Coolidge tube. This machine is designed to operate at 88,000 volts (peak) at a line voltage of 110–112. The voltage in our experiments was somewhat lower, since the line voltage approximated 104.[7] All treatments were given with a current of $2^3/_4$ m. a. passing through the tube, and at a target distance of 20 cm., measured to the center of the irradiated tassel. The treatments were applied to four series of 10 plants each, as follows:

SERIES	TREATMENT
I	1 exposure, 5 minutes
II	3 exposures, 2 minutes each, at 48-hour intervals
III	3 exposures, 5 minutes each, at 48-hour intervals
IV	3 exposures, 8 minutes each, at 48-hour intervals

Microscopic examination of daily pollen specimens showed that these treatments were severe enough to kill an appreciable proportion of the pollen. The pollen of untreated tassels usually contained at least 95% of good, well-filled grains, while that of the treated tassels, particularly those given the heavier treatments, contained a considerable proportion of empty or defective grains, reaching more than 50% in extreme instances. In many cases the injury was apparent during only a part of the pollen-shedding period, good pollen being produced both before and after. In the tassels receiving the two heavier treatments the number of anthers shedding pollen also was materially reduced. The untreated tassels of treated plants regularly produced good pollen.

The cross-over percentages in male gametes in treated and untreated tassels are shown in table 1. These data show some small but apparently significant differences in cross-over frequency. Treatment II shows a significantly lower percentage of *Sh-Wx* cross-over than the check in family

5009 and a significantly higher percentage in family 5017, and treatment I shows a significantly higher percentage in family 5001.

It may be shown, however, that these differences are not ascribable to the X-ray treatment. Since different plants within a family vary rather widely in crossing-over, considerable differences between small groups of treated and untreated plants may result from the accidental choice for treatment of plants normally high or low in crossing-over. In other words, there is a sampling error due to variation between plants, which is not measured in the probable errors determined from totalled progenies. The error in sampling plants may be avoided, however, by determining the effect of treatment only in comparisons of treated and untreated tassels

TABLE 1

EFFECT OF X-RAY TREATMENTS ON CROSSING-OVER

	CROSS-OVER PERCENTAGE			
	TREATMENT		UNTREATED	
TREATMENT	C-SH	SH-WX	C-SH	SH-WX
I (Family 5001)	4.33±0.24	23.84±0.27	4.69±0.24	21.80±0.47
II	2.87±0.24	14.81±0.48		
III	3.70±0.26	17.49±0.50	3.92±0.20	17.15±0.38
IV (Family 5009)	2.12±0.30	16.05±0.58		
II	4.25±0.23	24.27±0.49		
III	3.95±0.26	21.57±0.54	4.00±0.13	20.95±0.28
IV (Family 5017)	3.81±0.26	22.59±0.54		

of the same plants, for the earlier work has shown that different tassels of the same plant do not differ appreciably in cross-over frequency.

In 16 of the treated plants pollinations were made from both treated and untreated tassels. The difference in cross-over percentage was less than 3 times its probable error in all but 2 of the 16 cases for the *Sh-Wx* region, and in all but one case for the *C-Sh* region. In most of these cases daily pollinations were made in a search for effects of treatment which might last for only a part of the shedding period, but the results showed no grouping of high or low variations on successive days. These trials, therefore, show no appreciable effect of X-rays on cross-over frequency.

However, since the three "significant" differences just mentioned all occurred among the plants given the heaviest treatment, and since these three plants were slightly more advanced at the time of treatment than most of the other treated plants, a trial with heavier dosage and one more fully covering the maturation period seemed desirable. Accordingly, a second trial was made, in which the X-ray treatments were applied to the tiller tassels (which pass through the maturation stage much more quickly)

in daily applications beginning before maturation and continuing until after its completion. Four plants were given six exposures of five minutes each, four plants eight exposures of five minutes each, and two plants eight exposures of ten minutes each. Voltage, milliamperage and target distance were the same as before. In order to secure data for another chromosome region in the same trial, stocks were prepared heterozygous for *Sh-Wx* and also for *Y-Bh*, the only other endosperm linkage now available in maize. Unfortunately, it was found that in the stocks used the aleurone blotch *Bh* could not be distinguished accurately enough on starchy grains to permit critical determinations. The data from this trial, therefore, refer only to the *Sh-Wx* region.

The heavy treatments used in this trial greatly reduced the yield and viability of pollen in all cases. Only one treated tassel shed pollen more than two days. The amount shed was in most cases too small to permit the pollination of more than one or two ears per day. Microscopic examination of the pollen shed showed a very high proportion of defective grains, often higher than 50% and in some cases higher than 90%. The tassels given eight exposures of ten minutes each shed no pollen.

In seven of the ten plants treated, it was possible to determine cross-over frequency in both treated and untreated tassels. In each of these cases the difference in cross-over percentage was less than three times its probable error. The results give no indication of any effect of the X-ray treatment used on crossing-over in this region, or of any differential elimination of cross-over gametes by lethal doses of X-rays.

Chromosome Deficiency.—When plants recessive for an endosperm character are pollinated by the corresponding dominant, the heterozygous seeds produced include occasional mosaic individuals, in which a portion of the endosperm shows the recessive character. Emerson[8] has shown by genetic evidence that this is due, at least in large part, to aberrant chromosome behavior, since unlinked genes are lost independently while linked genes are almost always lost together. Because of the relative rarity of the phenomenon it has not been studied cytologically, and the nature of the process causing the deficiency is unknown. Non-disjunction and chromosome elimination fit the observed facts equally well. It is possible that gene mutation accounts for the small proportion of cases in which linked genes appear to be lost independently, but there are several possible explanations not yet excluded which may account for these cases without gene mutation.

To determine the effect of X-rays on this phenomenon the ears in which the frequency of mosaic endosperms was to be determined were X-rayed at or shortly before the time of fertilization, which, according to various observers, normally occurs about 24–28 hours after pollination. Voltage, milliamperage and target distance were the same as in the experiments on

TABLE 2

EFFECT OF X-RAY TREATMENTS ON THE FREQUENCY OF OCCURRENCE OF MOSAIC ENDOSPERMS

TRIAL	TREATMENT: NUMBER OF EXPOSURES	TREATMENT: DURATION OF EXPOSURES (MIN.)	TREATMENT: TOTAL TIME OF EXPOSURES (MIN.)	TREATMENT: INTERVAL BETWEEN EXPOSURES (HRS.)	TREATMENT: PERIOD FROM POLLINATION TO EXPOSURES (HRS.)		FREQUENCY OF MOSAICS* FOR CHARACTERS OF CHROMOSOME — I (C Wx, I)	II (R)	III (Sh)	V (Y)	VIII (Pr)	TOTAL FREQUENCY	TOTAL PERCENTAGE
I	1	5	5	–	24	Treated	1/135	0/122	2/417	1/245	0/58	4/977	0.41
						Untreated	0/273		0/294	0/185		0/752	0
	2	5	10	2	24–26	Treated		7/153	5/165	1/12	0/71	13/401	3.24
						Untreated	2/451	1/85	1/499	0/451	0/43	4/1529	0.26
II	3	10	30	3	21–27	Treated		19/314	16/447		0/132	35/893	3.92
						Untreated		2/599	1/675		1/336	4/1610	0.25
III	4	5	20	2	22–28	Treated	6/90	38/414	47/831		4/143	95/1478	6.43
						Untreated		3/507	0/703		0/126	3/1336	0.22
IV	5	5	25	2	20–28	Treated		18/262	8/297		8/297	34/856	3.97
						Untreated		0/385	1/395		0/174	1/954	0.10
Total			Frequency			Treated	7/225	82/1265	78/2157	2/257	12/701	181/4605	
						Untreated	2/724	6/1576	3/2566	0/636	1/679	12/6181	
			Percentage			Treated	3.11	6.48	3.62	0.78	1.71	3.93	
						Untreated	0.28	0.38	0.12	0	0.15	0.19	

* Including only mosaics in which the recessive character covered at least $^1/_8$ of the surface. The frequency is stated for each chromosome as total number of mosaics observed/total number of seeds in which a mosaic for this chromosome could have been detected.

crossing-over. In most cases intermittent exposures were given. Details of treatment are shown in table 2, together with the summarized results.

The four trials are not directly comparable, since they were made on different days and since several families were used in order to secure data for different chromosomes. Within each trial the treated and untreated groups were pollinated at the same time and from the same sample of pollen, and in most though not in all cases they include only ears of the same families. In determining the results mosaic endosperms showing the recessive characters on approximately one-half, one-quarter and one-eighth of the surface were recorded separately, since these presumably represent losses occurring at or before the first, second and third divisions, respectively. These have been summarized in preparing the table. Smaller spots were not included, since they cannot always be identified with certainty.

The results show a pronounced increase in the percentage of mosaic endosperms following X-ray treatment. It is possible that this effect may be due in part to an increase in the frequency of gene mutations as well as that of chromosome aberrations. The immense effect of X-rays on gene mutation in Drosophila, as recently reported by Muller,[9] suggests that this may be a factor. But unless the normal frequency of mutation during endosperm development is very much higher than that of the same genes previous to maturation,[10] even such extreme increases by X-ray treatment as those reported by Muller would not produce enough gene mutations to affect materially the percentage of mosaic endosperms. Unfortunately, direct evidence of the chromosomal nature of mosaics for chromosomes II, III and VIII, whether in treated or untreated material, cannot be obtained until linkages of endosperm genes in these chromosomes are found. Such evidence is obtainable for chromosomes I and V, but in these trials only a few mosaics for these chromosomes were included. The majority of these involved linked characters, and in all cases the linked genes were lost together, indicating that the mosaics in these cases at least were chromosomal.

The percentage of mosaics in the X-rayed series as a whole is more than 20 times as great as that in the untreated series. The frequency of mosaics for the five linkage groups varies somewhat, but since the data were obtained in part from different families, the significance of the differences is doubtful. With so high a frequency of aberrant divisions cases involving more than one chromosome would be expected, resulting in the simultaneous loss of unlinked genes. One such case was found in which the unlinked genes *I* and *Su* apparently were lost in the first division.

The frequency of the larger mosaics was increased by treatment more extremely than that of the smaller, indicating a rather rapid decrease in the effect of radiation. The frequency of "first division mosaics" was

2.56% in X-rayed seeds and 0.08% in untreated seeds. If these average frequencies, determined from five paternal chromosomes (but chiefly from chromosomes II and III), may be taken to represent roughly the frequency of loss of the 30 chromosomes present in the triploid endosperm nucleus, an average rate of about 75 aberrancies per 100 mitoses is indicated for the first division in endosperm development in the irradiated seeds.

[1] Formerly NATIONAL RESEARCH FELLOW. Part of the work reported was done under this fellowship at the Bussey Institution, Harvard University, 1925–26.

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GENERAL THEORY OF POLYGENIC OR NON-MONOGENIC FUNCTIONS. THE DERIVATIVE CONGRUENCE OF CIRCLES

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This paper is devoted mainly to the geometric questions connected with the differentiation of a general complex function $w = \varphi(x, y) + i\psi(x, y)$ with respect to the independent variable $z = x + iy$. We assume merely that the components φ and ψ are continuous and have continuous partial derivatives in the region considered, but do not assume the Cauchy-Riemann equations.

The limit of the increment ratio $\Delta w/\Delta z$, in general, then depends not only on the point $x + iy$ but also on the direction θ (or the slope m) along which the neighboring point approaches the given point. Thus the derivative dw/dz has, in general, infinitely many complex values at a given point. We call such a function *polygenic*. Only when the Cauchy-Riemann equations are fulfilled will the derivative have a unique value at the point; the function is then called *monogenic*. Thus the derivative of a polygenic function is a function of z and m while the derivative of a monogenic function is a function of z alone.

I show in the following that the ∞^1 values of dw/dz at each point z can be plotted as a circle and study the congruence of circles corresponding to the ∞^2 values of z. This congruence reduces to points only for monogenic functions.

I also introduce the new notion of *mean derivative*. It is used to define a certain class of harmonic transformations determined by polygenic functions whose components satisfy the Laplace equation without fulfilling the Cauchy-Riemann conditions.

Notation.—The notation will be as follows:

For the independent variable we use

$$z = x + iy,$$

for the dependent variable

$$w = u + iv,$$

and for the corresponding Gaussian planes (z) and (w). To indicate that w is a general polygenic function of z we write

$$w = \mathfrak{F}(z),$$

and reserve the familiar symbol

$$w = f(z)$$

for monogenic functions. Likewise

$$w = f(\bar{z})$$

shall denote an analytic function of $(x-iy)$ in the customary sense. Since the totality of polygenic functions contains the monogenic ones and the function of $\bar{z}$ as special cases, $\mathfrak{F}(z)$ can, under certain circumstances, specialize into $f(z)$ or $f(\bar{z})$.

For the real and imaginary components of $\mathfrak{F}(z)$ we take φ and ψ, so that

$$\mathfrak{F}(z) = \varphi(x, y) + i\psi(x, y).$$

For the sake of easier reference we insert here the Cauchy-Riemann equations and the corresponding conditions for functions of $(x-iy)$:

$$\begin{array}{ll} w = f(z): & w = f(z): \\ \varphi_x - \psi_y = 0 & \varphi_x + \psi_y = 0 \\ \varphi_y + \psi_x = 0 & \varphi_y - \psi_x = 0. \end{array} \qquad (1)$$

The transformation of (z) into (w) established by $w = \mathfrak{F}(z)$ will be called T; the transformation formulas are

$$\begin{aligned} u &= \varphi(x, y). \\ v &= \psi(x, y) \end{aligned}$$

If we differentiate w with respect to z we have

$$\frac{dw}{dz} = \frac{(\varphi_x + i\psi_x)\,dx + (\varphi_y + i\psi_y)dy}{dx + idy}.$$

We represent this expression, which is, of course, complex, by the symbol

$$\gamma = \alpha + i\beta.$$

Putting m for the slope dy/dx, we have finally

$$\gamma = \frac{\varphi_x + i\psi_x + m(\varphi_y + i\psi_y)}{1 + im}.$$

In the following we shall deal with the Gaussian planes of the three complex quantities

$$z = x + iy, \qquad w = u + iv, \qquad \gamma = \alpha + i\beta;$$

they will be abbreviated by (z), (w), (γ), and referred to as the first, second and third plane in this order. The third plane will frequently also be referred to as the derivative plane.

The Derivative Circle.—A polygenic function w of z sets up a correspondence between the first and third plane which is expressed by

$$\alpha + i\beta = \frac{\varphi_x + i\psi_x + m(\varphi_y + i\psi_y)}{1 + im}.$$

To a general point $x + iy$ belong ∞^1 points $\alpha + i\beta$, corresponding to the real parameter m. Since we are dealing with a linear function of m, these points are in one-to-one correspondence with the slopes m. For the same reason, the points must obviously form a circle. We call it the *derivative circle*. As we need its explicit equation in the following, we actually carry out the elimination of m and α and β. We have

$$\alpha = \frac{\varphi_x + m(\varphi_y + \psi_x) + m^2\psi_y}{1 + m^2}, \quad \beta = \frac{\psi_x + m(\psi_y - \varphi_x) - m^2\varphi_y}{1 + m^2} \tag{2}$$

$$\alpha - \psi_y = -\frac{1}{m}(\beta - \psi_x), \quad \alpha - \varphi_x = m(\beta + \varphi_y), \tag{3}$$

and so for the final equation

$$(\alpha - \psi_y)(\alpha - \varphi_x) + (\beta - \psi_x)(\beta + \varphi_y) = 0,$$

which may be rewritten

$$\left(\alpha - \frac{\varphi_x + \psi_y}{2}\right)^2 + \left(\beta - \frac{-\varphi_y + \psi_x}{2}\right)^2 = \left(\frac{\varphi_x - \psi_y}{2}\right)^2 + \left(\frac{\varphi_y + \psi_x}{2}\right)^2. \tag{4}$$

The ∞^1 points $\alpha + i\beta$ mapping the ∞^1 values assumed by the derivative dw/dz at a fixed point z form a circle; we call it the "derivative circle." Denoting by H and K its center coördinates and by R its radius, we have

$$H = \frac{\varphi_x + \psi_y}{2}, K = \frac{-\varphi_y + \psi_x}{2}$$

$$R = \tfrac{1}{2}\sqrt{(\varphi_x - \psi_y)^2 + (\varphi_y + \psi_x)^2}.$$

We notice that these three quantities are essentially made up of the expressions on the left-hand sides of the formulas (1).

We also remark that the derivative circle may degenerate into a point but never into a straight line.

The Congruence of Circles.—A polygenic function establishes a relation between the first and third plane in which to any point of the former corresponds a circle in the latter; to the ∞^2 distinct points of (z) will, therefore, in general correspond ∞^2 distinct circles in (γ). A configuration of ∞^2 curves is termed a congruence.

Thus the derivative of any polygenic function w is (in general) represented by a congruence of circles.

The limiting term "in general" in this theorem refers to "circles" as well as to "congruence;" it implies that degenerations in either respect are possible. Two very familiar cases furnish examples.

I. For monogenic function the derivative circles all reduce to points, since R vanishes identically (on account of the Cauchy-Riemann equations); and conversely if all the circles reduce to points the function must be monogenic.

II. For analytic functions of $(x-iy)$, H and K are identically zero and the double infinity of circles, therefore, reduces to a single infinity with the fixed center $(0, 0)$; conversely if the ∞^2 circles are to reduce to ∞^1 with the common center $(0, 0)$, w must be of the form $f(x-iy)$.

The whole congruence can also reduce to a single circle. We merely state this here, since we shall be able to give the discussion more simply in a later paper.

In general, any special property of a polygenic function is expressed by a corresponding property of the representative congruence; and vice versa, special types of polygenic functions are obtained by specifying special geometric types of the congruence. We give some further examples.

III. Determine all functions w such that at every point of (z) the modulus of the derivative is constant with regard to m. The derivative circles must then obviously reduce to points or have the fixed center $(0, 0)$. So by the converse theorems of I and II the required functions are of the forms $w = f(z)$ and $w = f(\bar{z})$.

IV. Similarly, the only functions for which the amplitude of the deriva-

tive at every point of (z) is independent of m are recognized, by the converse of I, to be the monogenic functions.

V. Require all the derivative circles to go through the origin. The analytic condition is found to be

$$\varphi_x\psi_y - \varphi_y\psi_x = 0,$$

that is, the required functions are characterized by the vanishing of the Jacobian of φ and ψ.

VI. Let the centers of all the circles lie on the α-axis. Then, as K vanishes, $-\varphi_y + \psi_x = 0$, so that

$$\varphi = W_x \quad , \quad \psi = W_y,$$

that is,

$$w = W_x + iW_y,$$

where W is an arbitrary function of x and y. Similarly, when all the centers lie on the β-axis, we find

$$w = -W_y + iW_x.$$

VII. Require all the centers to lie in a general fixed point. The corresponding functions are readily found to be

$$w = f(x - iy) + Ax + By + C,$$

where A, B, C are any complex constants.

We shall here introduce the term *affine* for the linear part of w, since later we will often have to deal with expressions of the same type. An "affine" then is an expression of the form

$$Ax + By + C,$$

where A, B, C are complex constants.

VIII. Represent z and γ in the same plane and require the points z and the centers of the respective derivative circles to coincide. The analytic condition is

$$\frac{\varphi_x+\psi_y}{2} = x, \quad \frac{-\varphi_y+\psi_x}{2} = y.$$

The complete solution is given by

$$w = \frac{z^2}{2} + f(x - iy).$$

The corresponding problem arising when w and γ are mapped in the same plane can also be solved in finite form; the result is

$$w = e^z f(x - iy).$$

The Ratio $(-2:1)$ *of the Angular Rates.*—After having become somewhat familiar with the congruence as a whole by the previous examples, let us now return again to a definite derivative circle corresponding to a definite point z and inquire how $\alpha + i\beta$ moves on the circle when m changes. The answer is given by the first formula of (3):

$$m = -\frac{\beta - \psi_x}{\alpha - \psi_y}$$

after verifying that the point $\psi_y + i\psi_x$ lies on the circle. For $\frac{\beta - \psi_x}{\alpha - \psi_y}$ is the slope of the line through $\psi_y + i\psi_x$ and $\alpha + i\beta$; according to the above equation, it is equal in magnitude and opposite in sign to the corresponding slope m. So we have a very simple method for constructing the point belonging to a certain m: it is the second intersection with the circle of the line of slope $-m$ through $\psi_y + i\psi_x$. If in this manner we construct two points P_1 and P_2 corresponding to the slopes m_1 and m_2, we see that the angle at the circumference over the arc P_1P_2 (since $\psi_y + i\psi^x$ lies on the circle) is equal in size to the angle between m_1 and m_2. Therefore, the cental angle over the same arc is twice as large as the original angle in (z), and we have the theorem:

When m changes, the point on the derivative circle moves so that its angular rate (measured on the circle) is twice that of m and in the opposite direction.

(It is obvious that on the circle the same point corresponds to two directions in (z) that differ by π; it is not obvious, however, that the ratio of the angular rates of m and the point on the circle should be constant.)

Thus each circle must be replaced by what I have called a "clock," and we must study *congruences of clocks*. See my paper in *Science*, vol. **66** (1927), pp. 281–282.

The Mean Derivative Operation $\mathfrak{D}$.—In order to be able to formulate some of the later results in a simple manner, we introduce a new term: we call the complex quantity $H + iK$ corresponding to the center of the derivative circle the *mean derivative* of w with regard to z and use for it the symbol

$$\mathfrak{D}(w).$$

Thus

$$\mathfrak{D}(w) = H + iK,$$

$$\mathfrak{D}(\varphi + i\psi) = \frac{\varphi_x + \psi_y}{2} + i\,\frac{-\varphi_y + \psi_x}{2}. \qquad (5)$$

The process of forming the mean derivative of $w = \psi + i\varphi$ consists

in forming the expression on the right-hand side of (5). Of course, $\mathfrak{D}(w)$ is also a polygenic function of z.

We also introduce the symbol $\mathfrak{D}^{-1}$; it stands for the operation inverse to that of forming the mean derivative. This inverse process gives a result which is determined not up to an additive constant but up to an additive term $f(x-iy)$.

The term "mean derivative" as introduced above actually agrees with the usual conception of "mean value." The mean value of $\alpha + i\beta$ considered as a function of m in a definite point z is in the customary sense if we put $m = \tan\theta$,

$$\frac{1}{\pi}\int_0^{\pi}(\alpha + i\beta)d\theta.$$

Carrying out the integration, *we actually obtain the expression on the right-hand side of* (5).

A polygenic function induces a certain point-transformation from the first plane to the third plane, namely that transformation in which to a point $x + iy$ of (z) corresponds the point $\mathfrak{D}(w)$ of (γ). Since this transformation leads from z to the center of the corresponding derivative circle, we call it the *induced center transformation;* we denote it by T'. Hence for any point-transformation T there is a related T'.

Harmonic Polygenic Functions and Harmonic Transformations.—The familiar theorem of the ordinary theory of functions that the derivative of a monogenic function is also monogenic, reads in the new terminology: when w is monogenic the induced center transformation is direct conformal. In symbols

$$\mathfrak{D}f_1(z) = f_2(z).$$

Let us put the converse question and ask whether

$$\mathfrak{D}^{-1}f_1(z) = f_3(z),$$

i.e., whether the monogenic functions are the only functions whose mean derivative is monogenic? If $H + iK$ is to be monogenic, then

$$H_x - K_y = 0 \qquad H_y + K_x = 0;$$

from which we find

$$\varphi_{xx} + \varphi_{yy} = 0 \qquad \psi_{xx} + \psi_{yy} = 0.$$

The necessary and sufficient condition to be fulfilled by $w = \varphi + i\psi$ in order that the induced center transformation may be direct conformal or, differently expressed, that the mean derivative of w may be monogenic, is that φ and ψ are harmonic functions.

We thus obtain a geometric interpretation for the case where the Laplace

equations are obeyed but not necessarily the Cauchy-Riemann equations.

According to familiar theorems, w then has the form

$$w = f_1(z) + f_2(\bar{z}).$$

We term this a *harmonic polygenic function*. The corresponding point transformation from the z-plane to the w-plane will be called a *general harmonic transformation*. The set of all such transformations does not form a group but it contains the group of all conformal transformations (direct and reverse) as a part.

Forthcoming papers on polygenic functions will appear in *Comptes Rendus* and *Bull. Amer. Math. Soc.*

NUMBER OF SYSTEMS OF IMPRIMITIVITY OF TRANSITIVE SUBSTITUTION GROUPS

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It is well known that the number of the systems of imprimitivity of a regular substitution group is equal to the number of its proper subgroups, that is, the number of its subgroups excluding the identity and the entire group. This is a special case of the following general theorem: *The number of the systems of imprimitivity of any transitive substitution group is equal to the number of the proper subgroups of this group which separately involve as a proper subgroup a fixed one of the subgroups composed of all the substitutions of this transitive group which omit a given letter, and the number of letters in a set of such a system of imprimitivity is equal to the index of this fixed subgroup under the larger subgroup.* To prove this theorem it is only necessary to observe that if G_1 is composed of all the substitutions of a transitive group G which omit a given letter, and if G_1' is any proper subgroup of G which involves G_1 as a proper subgroup, then the letter which is omitted by all the substitutions of G_1 is transformed into a certain number of conjugates under G_1'. Moreover, G_1' involves all the substitutions of G which transform these letters among themselves since it involves all of these substitutions which transform the given omitted letter into itself. Hence these letters constitute a set of a system of imprimitivity of G.

On the other hand, when G is imprimitive, such a subgroup G_1' must exist for every possible system of imprimitivity since all the letters of G must appear in every one of its possible systems of imprimitivity. Each of the subgroups G_1' must, therefore, characterize the system of imprimitivity to

which it belongs. When G is regular G_1' is any proper subgroup of G since G_1 reduces to the identity in this special case and hence it is found in every subgroup. When G is the group obtained by extending a regular abelian group involving at least one substitution whose order exceeds two by means of a substitution of order 2 on its letters which transforms into its inverse every substitution of this regular group, then the resulting group has one and only one system of imprimitivity for every proper subgroup of this regular group according to the given theorem and the number of letters in a set of a system of imprimitivity is equal to the order of this subgroup. The number of its systems of imprimitivity is, therefore, always exactly the same as the number of these proper subgroups. In particular, when the regular non-cyclic group of order p^2 is thus extended, there results a group which has exactly $p + 1$ different systems of imprimitivity and the number of letters in a set of each of these systems is p.

Since every proper subgroup of G which includes G_1 is intransitive it results that a necessary and sufficient condition that G_1 is a maximal subgroup of G is that G_1 is not a proper subgroup of an intransitive subgroup of G. When G_1 is transitive and its degree is just half of the degree of G then it is known that G_1 is regular and that G involves the direct product of G_1 and its conjugate. Hence the number of proper subgroups of G which involve G_1 as a proper subgroup is equal to one more than the number of proper subgroups of G_1. This is also the number of the systems of imprimitivity of G, according to the given theorem. Hence the following additional theorem has been established. *If in a transitive substitution group G the subgroup G_1 composed of all the substitutions of G which omit a given letter is transitive and omits half the letters of G, then the number of the systems of imprimitivity of G is one more than the number of the proper subgroups G_1.*

When G_1 is transitive and omits just half of the letters of G it is easy to see that every pair of the letters of G_1 appears in at least one set of a system of imprimitivity of G, but no pair composed of a letter of G_1 and a letter not found in G_1 appears in such a set. On the other hand, in the case of all the groups considered in the second paragraph every pair of letters obviously appears in at least one set of some system of imprimitivity. When G_1 is intransitive and omits exactly half of the letters of G, then G again involves an intransitive subgroup of index 2, whose two transitive constituents are conjugate and regular, and the number of the different systems of imprimitivity of G in which the separate sets involve either only letters of G_1 or only letters not contained in G_1 is again one more than the number of the proper subgroups of one of these two given conjugate regular groups. A necessary and sufficient condition that such a G contains any additional system of imprimitivity is that its quotient group with respect to the direct product of G_1 and its conjugate is non-cyclic.

When this condition is satisfied the number of additional systems of imprimitivity is equal to the number of the proper subgroups of this quotient group which are such that each of them involves at least one operator which does not correspond to the given intransitive subgroup of index 2 under G. Hence it results directly that if there is one such additional system of imprimitivity, there are at least two. What precedes gives, therefore, a complete determination of the possible systems of imprimitivity of a group, in which the subgroup composed of all the substitutions which omit one letter omits exactly half the letters of G.

It may be added that when G contains invariantly the direct product of k conjugate transitive groups, but no substitution which transforms into itself one of these transitive groups unless its constituent on the letters of this transitive group appears therein, then the number of the systems of imprimitivity of G is just one larger than the number of the systems of imprimitivity of each of the given transitive groups increased by the number of the systems of imprimitivity in the transitive group of degree k, according to which the factors of the given direct product are transformed under G. If this group of degree k is primitive G involves, therefore, just one more system of imprimitivity than one of the k constituents of the given direct product. Hence this method enables us to construct a non-abelian group which has any desired number of systems of imprimitivity. Since the cyclic group of order p^α, p being a prime number, has exactly $\alpha-1$ proper subgroups, it results that such a group has exactly $\alpha-1$ systems of imprimitivity when it is represented as a regular group, and hence it is possible to construct a regular cyclic group which also has any desired number of systems of imprimitivity. An infinite system composed of groups having separately just two systems of imprimitivity may be obtained by extending a simple isomorphism between two symmetric groups of degree $n > 2$ by means of a substitution which is commutative with every substitution in this intransitive group.

If G_1 has the property that the group generated by it and any arbitrary substitution of G is a proper subgroup of G, then it results from what was noted above that a letter of G omitted by all the substitutions of G_1 appears in a set of a system of imprimitivity of G, such that this set involves an arbitrary other letter of G. Hence, the following theorem has been established: *A necessary and sufficient condition that every pair of letters of G appears in at least one set of a system of imprimitivity of G is that G_1 and an arbitrary substitution of G which is not found in G_1 always generate a proper subgroup of G.*

CONCERNING TRIODS IN THE PLANE AND THE JUNCTION POINTS OF PLANE CONTINUA

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Menger[1] and Alexandroff[2] use the term *verzweigungspunkt*, and Kuratowski and Zarankiewicz[3] the term *point of ramification*, to denote a point of Menger order greater than two of a regular curve. It has been shown, by Menger,[4] that if P is such a point of a regular curve C then C contains three simple continuous arcs such that P is common to all three of them and such that no two of them have any other point in common.

Wazewski[5] and Menger[1] have shown that the set of all the points of ramification of an acyclic[6] continuous curve is countable and Alexandroff[2] and Menger[1] have established the same result for a finitely connected continuous curve, that is to say for a continuous curve that does not contain more than a finite number of simple closed curves. It is clear, however, that this proposition does not[3] hold true for all regular curves.

In the present paper I shall introduce the notion *junction point of a continuum* and shall establish the theorem that no plane continuum whatsoever has more than a countable number of junction points. In the presence of the easily verified fact that every point of ramification of a regular curve of finite connectivity is also a junction point of that curve, the above-mentioned results of the authors cited are (insofar as they apply to the plane) special cases of this much more general theorem.

DEFINITION.—If O, A_1, A_2 and A_3 are four distinct points, and for each n $(1 \leqq n \leqq 3)$, r_n is an irreducible continuum from A_n to O and no two of the continua r_1, r_2 and r_3 have, in common, any point except O then the continuum $r_1 + r_2 + r_3$ is a *triod*, the point O is an *emanation point*, and the continua r_1, r_2 and r_3 are *rays* of this triod.

It may be easily shown that no triod has more than one emanation point or more than one set of three rays. It is, therefore, permissible to speak of *the* emanation point and *the* rays of a given triod. If O is a point and M_1, M_2 and M_3 are three continua which have O in common and no two of them have in common any point except O then the continuum $M_1 + M_2 + M_3$ contains at least one triod whose emanation point is O.

LEMMA 1.—*If, in space of two dimensions, J is a simple closed curve and t_1, t_2 and t_3 are three mutually exclusive arcs lying on J, and H and K are triods which are subsets of the point set composed of J plus its interior and h_1, h_2 and h_3 are the rays of H, and k_1, k_2 and k_3 are the rays of K and, for each n $(1 \leqq n \leqq 3)$, t_n contains a point of h_n and a point of k_n, then the triods H and K have at least one point in common.*

Lemma 1 may be proved with the help of various theorems established by Janiszewski,[7] Rosenthal[8] and Miss Mullikin.[9]

THEOREM 1.—*If, in space of two dimensions, G is an uncountable set of triods, there exists an uncountable subset of G such that every two triods of this subset have a point in common.*

Proof.—There exists a positive number d and an uncountable subset G_1 of G such that all three rays of every triod of the set G_1 are of diameter greater than d. Let O denote a point which is an emanation point of some triod of the set G_1 and a point of condensation of the set of all the points of emanation of the triods of G_1. Let K denote a circle with center at O and a radius equal to $d/2$. Let G_2 denote the set of all those triods of G_1 whose emanation points lie within K. For each triod t of the set G_2 select a definite triod q_t with the same emanation point as t and such that (a) q_t is a subset of t, (b) each ray of q_t is[9] an irreducible continuum from the emanation point of t to K. Let Q denote the set $[q_t]$ for all triods t of G_2. For each triod q of the set Q select three definite points A_q, B_q and C_q, one on each ray of q, and all lying on K. There exists a positive number e and an uncountable subset Q_1 of Q such that if q is any triod of the set Q_1 then every two of the points A_q, B_q and C_q are at a distance apart greater than e. It follows that there exist, on K, three distinct points A, B and C such that if x is any positive number there are uncountable many triods q belonging to Q_1 and such that the points A_q, B_q and C_q are at distances less than x from the points A, B and C, respectively. Let a, b and c denote three mutually exclusive arcs of K with midpoints at A, B and C, respectively. The set Q_1 contains an uncountable subset Q_2 such that if q is any triod of the set Q_2 then A_q, B_q and C_q belong to a, b and c, respectively. Every triod of the set Q_2 is a subset of K plus its interior. It follows, by Lemma 1, that every two triods of the set Q_2 have a point in common.

THEOREM 2.—*If, in space of two dimensions, G is an uncountable set of mutually exclusive bounded continuous curves then all but a countable number of them are either simple continuous arcs or simple closed curves.*

Theorem 2 may be easily proved with the aid of theorem 1 and the fact that if a bounded continuous curve contains no triod then it is either a simple continuous arc or a simple closed curve.

THEOREM 3.—*There exists, in space of two dimensions, an uncountable set G of mutually exclusive bounded continua such that no continuum of the set G contains an arc.*

Proof.—Knaster[10] has shown that there exists an indecomposable continuum which contains no decomposable subcontinuum. Let M denote such a continuum. By a theorem of Janiszewski and Kuratowski's,[11] M has an uncountable number of different composants. For each com-

posant of M select just one continuum which is a subset of that composant. The set of continua so obtained is uncountable, no two of them have a point in common, and no one of them contains an arc.

THEOREM 4.—*If, in space of two dimensions, G is an uncountable set of mutually exclusive continuous curves then all but a countable number of them are simple*[12] *continuous curves.*

DEFINITION.—A point P of a continuum M is said to be a *junction point* of M if it is the emanation point of some triod which is a subset of M and there exists a domain D containing P and such that P is a cut point of the greatest connected subset of $M \cdot D'$ that contains P.

LEMMA 2.—*If P is a cut point of a continuum M and D is a domain containing P and H is a connected subset of M containing the greatest connected subset of $M \cdot D$ that contains P, then P is a cut point of H.*

The truth of this lemma may be easily established. It does not remain true if "continuum M" is replaced by "connected point set M."

THEOREM 5.—*No continuum has uncountably many junction points.*

Proof.—Suppose, on the contrary, that there exists a continuum M with an uncountable number of junction points. Then, as may be seen with the help of lemma 2, there exists a positive number d and an uncountable set K of junction points of M such that if P is a point of K and I_P is the interior of a circle with center at P and a radius equal to d and M_P is the greatest connected subset of $M \cdot I'_P$ that contains P then P is a cut point of M_P and M contains a triod having P as its emanation point. Let O denote a point of K which is a point of condensation of K and let D_O denote the interior of a circle with center at O and radius equal to $d/4$. For every point X of K lying in D_O let D_X denote the interior of a circle with center at X and radius equal to $d/4$ and let N_X denote the greatest connected subset of $M \cdot D'_X$ that contains X. With the help of theorem 1 it may be easily shown that the point set $K \cdot D_O$ contains an uncountable subset L such that if X and Y are any two points of L then N_X and N_Y have a point in common. Let P denote any given point of L. The domain I_P contains N_X for every point X of L. Thus the sum N of all the sets N_X for all points X of L is a connected subset of I_P. Furthermore, N contains P. Hence, it is a subset of M_P. Thus, for every point P of L, $N + N'$ contains N_P and is a subset of M_P. It follows, by lemma 2, that every point of L is a cut point of $N + N'$. Furthermore, no point of L is a point, of Menger order two, of $N + N'$. But, by a theorem of G. T. Whyburn's,[13] no continuum has uncountably many cut points that are not of Menger order two. Thus the supposition that theorem 5 is false has led to a contradiction.

[1] Menger, K., "Ueber reguläre Baumkurven," *Math. Ann.*, **96**, 1926 (572–582).

[2] Alexandroff, P., "Ueber kombinatorische Eigenschaften allgemeiner Kurven," *Math. Ann.*, **96**, 1926 (512–554).

[3] C. Kuratowski and C. Zarankiewicz, "A Theorem on Connected Point Sets," *Bull. Amer. Math. Soc.*, **33**, 1927 (571–575).

[4] Menger, K., "Zur allgemeiner Kurventheorie," *Fund. Math.*, **10**, 1927 (96–115).

[5] Wazewski, T., "Sur les courbes de Jordan ne renfermant aucune courbe simple fermee de Jordan," *Annales de la Société Polonaise de Mathématique*, **2**, 1923 (49–170).

[6] A continuous curve is said to be acyclic if it contains no simple closed curve. Cf. Gehman, H. M., "Concerning Acyclic Continuous Curves," *Trans. Amer. Math. Soc.*, **29**, 1927 (553–568).

[7] Janiszewski, S., "Sur les coupures du plan faites par les continus" (en polonais), *Prace matematyczno-fizyczne*, **26**, 1913 (11–63).

[8] Rosenthal, A., "Teilung der Ebene durch irreduzible Kontinua," *Münchener Akademie, Sitzungsber.*, **1919** (91–109).

[9] Mullikin, Anna M., "Certain Theorems Relating to Plane Connected Point Sets," *Trans. Amer. Math. Soc.*, **24**, 1922 (144–162).

[10] Knaster, B., "Un continu dont tout sous-continu est indécomposable," *Fund. Math.*, **3**, 1922 (247–286).

[11] Janiszewski and Kuratowski, "Sur les continus indécomposables," *Fund. Math.*, **1**, 1920 (210–222).

[12] A continuous curve is said to be simple if it is either an arc, an open curve, a simple closed curve or a ray of an open curve. See my paper "Concerning Simple Continuous Curves," *Trans. Amer. Math. Soc.*, **21**, 1920 (333–347).

[13] Whyburn, G. T., "Concerning the Cut Points of Continua." Presented to the American Mathematical Society, Sept. 9, 1927, but not, as yet, published in full.

ON METHODS AND APPLICATIONS IN SPECTROPHOTOMETRY

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1. The interpretation of stellar spectra is not possible without precise photometry. Just as it was necessary that stellar spectra should be photographed before they could be discriminatingly classified, it is essential that the photographed spectra be measured before they can be interpreted in the light of modern theory. The inadequacy of eye estimates for photographic photometry has repeatedly been demonstrated and, accordingly, recent spectroscopic work at Harvard has been largely concerned with the development and standardization of methods of photographic spectrophotometry by means of the Moll microphotometer. The present note is intended to summarize the results of two years' experiments.

The basic problem of the calibration of a photographic plate involves the determination of the blackening that corresponds to a given intensity of incident light. As the photographic plate responds selectively to light of different wave-lengths, and as the nature of this response varies for different emulsions, a complete standardization for any one emulsion

would include an intensity-blackening curve for every wave-length.

But stellar spectrophotometry at present cannot usefully be made anything but relative; the selective absorption suffered by starlight in traversing both the atmosphere and the optical train of the telescope makes

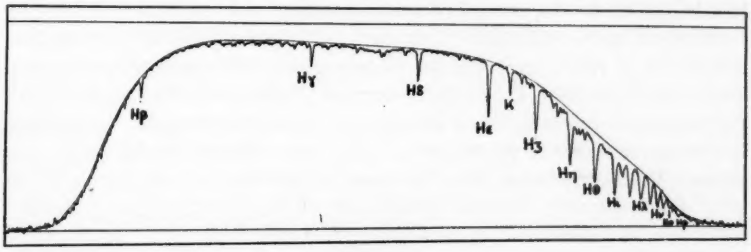

FIGURE 1

Microphotometer tracing from the spectrum of α Cygni. Horizontal lines above and below the tracing represent "darkness" and "clear film." The continuous background is indicated by a light line. The lines of the Balmer series and of Ca^+ are marked.

it impractical to attempt absolute measures. The two fundamental measures that can be made on stellar spectrograms—the absorption at any wave-length and the relative background energy—must both be made differentially; the one is related to the neighboring continuous background, graphically inserted (see Fig. 1), and the other to the background of another star of known energy distribution—both the measures to be compared being made *at the same wave-length*. To compare intensities in this way it is only necessary to know the gradation of the plate in

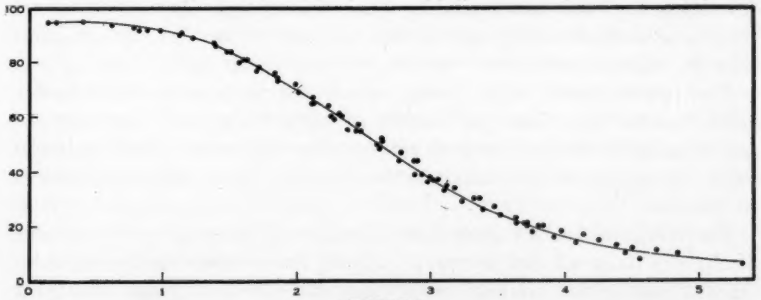

FIGURE 2

Curve for reducing galvanometer deflections (ordinates) to stellar magnitudes (abscissae). Adapted from Harvard Observatory Bulletin 301.

every wave-length; the absolute intensity-blackening relation is irrelevant.

Very fortunately it appears, from numerous tests made in the course of the work now described,[1] that the gradation of the plates used, under the standard conditions, does not vary sensibly with wave-length, at

least within the region usually photographed. Therefore, one reduction curve, relating increase of blackening to increase of intensity, can be used for the whole length of the spectrum on each plate. It is to this experimental approximation that the possibility of rapid and comprehensive methods of spectrophotometry is due.

2. Various methods have been used for the establishment of reduction curves, all of them involving the illumination of the plate, either successively or simultaneously, by light sources of known intensity ratio. All the satisfactory methods use an exposure of uniform length. According to the several ways of governing the differences of intensity of the various sources, the methods used may be classed as follows:

METHOD	REFERENCE	
(*a*) Various diaphragms on one star	Shapley	*Harv. Obs. Bul.* 805
	Payne and Shapley	*Harv. Repr.* 28
	Payne and Hogg	*Harv. Obs. Circ.* 301
(*b*) Several stars of known brightness, same spectral class	Payne and Hogg	*Harv. Obs. Circ.* 301
(*c*) Several stars of known brightness, different spectral classes	Hogg	*Harv. Obs. Circ.* 309
(*d*) Differences within same spectrum	Payne	Unpublished
(*e*) Laboratory standardization	Dunham	*Harv. Obs. Bul.* 853
(*f*) Exposure ratios on one star	Payne and Hogg	*Harv. Obs. Circ.* 304
(*g*) Prism crossed with grating	Hogg	Unpublished

The methods are further described in the papers quoted and will not be discussed in detail here. It will be noticed that for slit spectra methods (*d*), (*e*) and (*f*) only are available; (*f*) is, however, quite unsatisfactory. The three methods chiefly used at Harvard, (*a*), (*b*) and (*c*), are available only in connection with the objective prism spectrograph.

The prism crossed with grating, which has been used at Greenwich with success,[2] has been the subject of some preliminary experiments, and is probably the best method for very accurate contour work on bright stars. It is proposed to use it in the analysis of long dispersion spectra of the sun.

There is little doubt that method (*c*) will eventually replace the others, because of its speed and accuracy. As the two or more spectra used for standardization are simultaneously photographed, it eliminates changing sky conditions and corrections arising from differential refraction, two factors which may cause serious error for apertured spectra when the stars photographed are faint. Work has been begun on a program of standard pairs of stars, suitably distributed around the sky.

Measurement of the blackening of the plates has been made with the Moll self-registering microphotometer for all the researches discussed. The Schilt visual microphotometer has, however, obvious applications,

especially to the measurement of short dispersion spectra, and with it unpublished results have already been derived.

3. When the photographic plate has been calibrated, so that the relation of light intensity to density of image is known, it remains to select significant quantities for measurement. The measurable quantities are evidently the blackness and total absorption of the lines, and the relative energy distribution in the continuous background.

The nature of the quantity measured by the "blackness" or central intensity is discussed by one of the writers in *Harvard Circular* 302, where it is shown that for such wide lines as the H and K lines of ionized calcium in cool stars the central intensity represents the true "residual intensity" of the line, and for the narrow lines it represents the total absorption. The resolving power of the spectrograph affects all measured depths; therefore, only for spectra made with the same instrument can depths be usefully compared. In addition to instrumental effects there are doubtless disturbances arising in the atmosphere of the earth, and possibly also in the atmosphere of the star,[3] which modify the residual intensity, increasing it beyond what would be observed if the theory of the high level chromosphere[4] applied to it rigorously.

The relatively small dispersion of the Harvard spectrograms results in the blending of some important lines. Only lines that are very intense and undoubtedly predominant are included in the discussions, and the possible effects of blending are always kept in mind.

On the other hand, the total absorption of a line is probably unaffected by the resolving power of the spectrograph, and may, therefore, be measured by suitably integrating the line contour. A refined method of doing this is described by Dunham[5] and one suited to more rapid work, by one of the writers.[6] Total energy of lines has an obvious significance, both for stars with absorption spectra, and for emission line objects, such as novae in their later stages, Wolf-Rayet stars, and notably gaseous nebulae. Measures for such objects are now being made at Harvard Observatory.

The determination of the relative energy distribution in the continuous background results from a simple differential comparison with the standard star at various wave-lengths, and may be directly used in the measurement of relative temperature.

4. The results hitherto attained with the aid of the methods enumerated are summarized in the next table. The two main measured parameters, line intensity and relative background energy are the material, respectively, for the spectrum analyses and the measured temperatures.

The researches cover most of the spectral sequence, and are of general as well as special interest. We shall note first a few points brought out by special researches, and then touch lightly on some general implications of the material—implications that must be regarded as preliminary and tentative.

The study of the eight brighter Pleiades shows essential similarity for the six stars of class B5 in the intensities of the Balmer lines. Merope, the most deeply enmeshed in nebulosity, has, however, the shallowest line and the most conspicuous hydrogen emission, and is considerably the reddest of the B5 stars. The emission lines of Alcyone, which are also conspicuous, are probably an accompaniment of its greater luminosity, and not related to the nebula. The relative and actual intensities of the hydrogen lines change with the spectral class, Atlas and Pleione (Class B8) having deeper lines than the other six stars. Definite criteria of luminosity for stars of class B5 are not found.

Mira Ceti was followed from the maximum of 1926 until it fell below visual magnitude 6.5. At maximum the hydrogen lines, which are then at their strongest, contributed not more than a tenth of a magnitude to

	STARS	MEASURES	REFERENCE
(*a*)	Pleiades	Spectrum analysis	*Harv. Obs. Circ.* 303
(*b*)	Pleiades	Temperatures	*Harv. Obs. Circ.* 309
(*c*)	Sirius and Vega	Spectrum analysis	*Harv. Obs. Circ.* 304
(*d*)	Fourteen A stars[5]	Spectrum analysis	*Harv. Obs. Bul.* 853
(*e*)	Nine second type stars	Spectrum analysis	*Harv. Obs. Circ.* 305 and 306
(*f*)	Mira Ceti	Spectrum analysis	*Harv. Obs. Circ.* 308
(*g*)	Wolf-Rayet stars	Temperatures	*Harv. Obs. Bul.* 848
		Spectrum analysis	Unpublished
(*h*)	Mizar	Spectrum analysis	Unpublished
(*i*)	δ Cephei and η Aquilae	Spectrum analysis	Unpublished
(*j*)	Class M stars	Spectrum analysis	Unpublished
(*k*)	High luminosity stars	Spectrum analysis	Unpublished
(*l*)	Standard fields	...	Unpublished

the photographic light of the star—and considerably less to the visual brightness, for Hα is not a strong emission line. The titanium oxide bands, over the interval examined, divert about twenty per cent of the star's photographic light. These results, derived over a part of the period at which the light of the companion is not conspicuous, are probably typical of many long period variable stars.

The estimates of temperature published for the O stars near the cluster N. G. C. 6231 have probably little physical meaning, as it is shown that the energy in the background curves does not correspond to a black body distribution. "Color magnitudes," instead of black body temperatures, are proposed as a better way to express the energy distribution determined spectrophotometrically. The abnormal distribution of energy, and the surprisingly low temperatures found for the northern Wolf-Rayet stars, are borne out by further studies, now in progress, of the spectra of southern stars of the class.

Turning to the general conclusions that can be drawn from the material,

it is found that the behavior of integrated line areas along the spectral sequence is roughly parallel to that of the absorption lines, as estimated visually. The observed maxima discussed by ionization theory[7] are no doubt maxima of total absorption. For lines that are wide enough for the measures to be significant, it appears that central intensity for a given line passes through a minimum that coincides approximately with the maximum of total absorption, though there is a gradual change of line contour in passing along the spectral sequence, so that the connection is not quite a simple one. These facts place the Saha-Fowler-Milne theory for the first time on a quantitative basis, and also, as Milne has pointed out, make it possible to develop a theory of atmospheric structure.

The relation of line contour or quality to luminosity within a given spectral class has long been known, and measures such as the present contain the material for placing it too on a quantitative basis. It is found that the total absorption of all lines is greater for intrinsically bright stars, and that contours are more flattened for fainter stars, effects that have a definite empirical use, and possibly a theoretical one, in the discrimination and interpretation of luminosities. The observed intensities of the H and K lines have also an immediate application to the study of the presence and structure of chromospheres.

Some of the matters mentioned in the present section are discussed in *Harvard Circular* 307; the remainder await the publication of the data relating to the supergiant stars, and the discussion of the spectra of the Cepheid variables, which will be used to calibrate the scale by which the other highly luminous stars are to be examined.[8]

Summary.—1. The problem presented by stellar spectrophotometry is briefly surveyed.

2. The methods applied at Harvard to its solution are enumerated and compared.

3. The quantities that can be usefully measured in stellar spectra are discussed.

4. The subjects of recent spectrophotometric studies are tabulated, and the paper concludes with a short account of some special researches, and with a general interpretation of material now available.

[1] Payne and Hogg, *Harv. Obs. Circ.* 301, 1927.
[2] Greaves, Davidson and Martin, *Mon. Not. R. A. S.*, **86**, 1925; *Ibid.*, **87**, 1927.
[3] Eddington, *The Internal Constitution of the Stars*, **1926**, p. 342.
[4] Milne, *Mon. Not. R. A. S.*, **84**, 1924; *Ibid.*, **85**, 1925.
[5] Dunham, *Harv. Obs. Bul.* 853, 1927.
[6] Payne, *Harv. Obs. Circ.* 302, 1927.
[7] Payne, *Harv. Obs. Monograph*, No. 1, **1925**, pp. 116–132.
[8] Shapley, *Harv. Obs. Circ.* 313, 1927.

THE EFFECT OF POSTURE UPON THE COMPOSITION AND VOLUME OF THE BLOOD IN MAN

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Measurements have been made to determine the composition and volume of the blood in man in the recumbent and in the standing still positions. These measurements are summarized in table 1.

It may be seen that during the period of standing still,[3] after a rest in the recumbent position, there are the following changes in the blood:

1. An increase in the number of red cells per cu. mm. of blood (I).
2. A corresponding increase in the volume of red cells per liter of blood (II).
3. An increase in the concentration of plasma protein (IV).
4. An increase in the specific gravity of plasma (VI).
5. A decrease in the concentration of plasma water (V).
6. A decrease in the total amount of plasma in the blood of the whole body (III).

Observations on the mixing of a dye, according to a slight modification (1) of the usual method employed in estimating plasma volume, have been made and are summarized on a typical case in tables 2 and 3. These data show two significant things:

1. A much longer time is required for the dye concentration in the blood to become uniform all over the body in the standing still position than in the recumbent position.
2. When the concentration has become uniform, it is always greater in the standing still position than in the recumbent position.

Confirmatory evidence of the effect of posture upon plasma volume may be found in the hematocrit measurements of cell volume recorded in table 4.

From these observations, it seems safe to draw the conclusion that, while standing still, there is temporarily a much greater loss of fluid from the blood than gain to it through the thoracic duct and other sources.

Assuming that the fluid lost is protein free, the changes in red cell count, plasma protein and plasma water calculated from the changes in total plasma volume correspond well with the actual observations (table 5). Moreover, the total cell volume for each subject, calculated from the total plasma volume and the cell percentage, is the same in both the recumbent and standing still positions (table 6).

The reciprocal relation between the protein and water changes is shown in figure 1.

These facts are consistent with the hypothesis that there has been on the average a net loss of 110 cc. of protein free fluid per liter of plasma, or a total loss of 290 cc., and no other change.

Upon resuming the recumbent posture the changes are reversed. The maximum fluid loss, which occurs in the standing still position in 20–30 minutes, is made up in about the same time in the recumbent position.

Most of the changes in the composition of the blood here noted are known to occur in the venous blood of an extremity as a result of raising the venous pressure (2), (3), (4), (5). Moreover, it has been demonstrated that, in

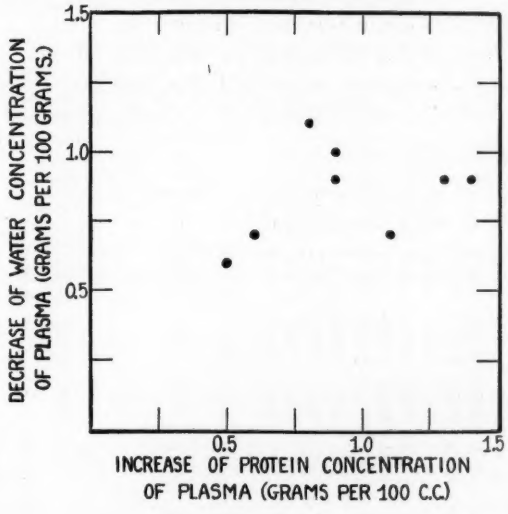

FIGURE 1

Increase in plasma protein concentration and decrease in plasma water concentration following change from the recumbent to the standing still position. (Table 1.)

the standing still position, the return of blood from dependent portions of the body is markedly retarded, (6), (7), and that the venous and capillary pressures in these parts are raised (8), (9), (10). These facts, already known, seem to offer a satisfactory explanation of the present observations.

Conclusion.—In the standing still position there occurs a net loss of approximately protein-free fluid from the blood. This seems to be due to an increase in capillary pressure. The loss amounts, on the average, to about 11 per cent of the total plasma volume, and is probably greatest where the filtration pressure is most increased, viz., in the lower extremities.

TABLE 1

CASE NO.	AGE	SEX	I RED BLOOD CELL COUNT (MLNS./CU. MM.) LYING	I STANDING	I DIFF.	II CC. OF CELLS PER 100 CC. OF BLOOD LYING	II STANDING	II DIFF.	III TOTAL PLASMA VOLUME (CC.)* LYING	III STANDING	III DIFF.	IV PROTEIN (GRMS./100 CC.) LYING	IV STANDING	IV DIFF.	V PLASMA WATER (GRMS./100 GRMS.) LYING	V STANDING	V DIFF.	VI SPECIFIC GRAVITY LYING	VI STANDING
1	55	♂	4.8	5.7	+0.9	44.6	49.6	+5.0	3055	2580	−475								
2	60	♂	4.4	4.4	0.0	45.9	48.8	+2.9	2770	2445	−325								
3	38	♀	4.1	4.6	+0.5	40.1	44.2	+4.1	2140	1900	−240	7.2	8.3	+1.1	91.1	90.4	−0.7	1026	1029
4*A*†	37	♀	3.9	4.3	+0.4	35.6	38.0	+2.4	2495	2195	−300	6.9	8.2	+1.3	91.5	90.6	−0.9	1026	1027
5	31	♂	4.7	5.1	+0.4	47.8	50.9	+3.1	2505	2160	−345	7.0	7.8	+0.8	91.9	90.8	−1.1	1025	1029
6	43	♀	4.1	4.6	+0.5	28.1	30.2	+2.1	2755	2485	−270	6.5	7.9	+1.4	91.3	90.4	−0.9	1028	1029
7	29	♀							1810	1580	−230	6.1	7.0	+0.9	92.2	91.3	−0.9		
8	28	♂							3200	2985	−215	7.0	7.5	+0.5	91.3	90.7	−0.6		
4*B*	37	♀							2495	2305	−190	7.2	7.8	+0.6	91.2	90.5	−0.7		
9	59	♂							2835	2525	−310	7.3	8.2	+0.9	91.0	90.0	−1.0		
Average for cases 1–6 inclusive			4.3	4.8	+0.5	40.3	43.6	+3.3	2620	2295	−325								
Average for cases 3–6 inclusive			4.2	4.7	+0.5	37.9	40.8	+2.9	2475	2185	−290	6.9	8.1	+1.2	91.5	90.6	−0.9	1026	1028
Average for cases 3–9 inclusive												6.9	7.8	+0.9	91.4	90.6	−0.8		
Average plasma volumes for all cases									2605	2315	−290								

Nearly every determination recorded is the average of two or more measurements.

† Case numbers 4*A* and 4*B* refer to two distinct sets of experiments on the same subject.

* In cases 1 to 6, inclusive, all plasma volumes were determined by the dye method. In cases 7, 8, 4*B* and 9, the plasma volume for the standing still position was calculated from the change in protein concentration. In cases 7 and 8, it was arbitrarily assumed to be 40 cc. per kg. for the recumbent position, and in cases 4*B* and 9 it was measured for this position by the dye method.

TABLE 2

EFFECT OF POSTURE UPON THE CONCENTRATION OF A DYE INJECTED INTRAVENOUSLY (Case 1)

POSITION OF BODY	AVERAGE CONCENTRATION OF DYE* IN PLASMA AFTER COMPLETE MIXING: RIGHT ARM VENOUS BLOOD %	LEFT ARM VENOUS BLOOD %	FOOT VENOUS BLOOD %
Lying down	101.1	99.9	99.6
Standing still	119.1	116.9	120.4

* The average plasma volume for the recumbent position, viz., 3055 cc., has arbitrarily been chosen to represent 100 per cent concentration of dye.

TABLE 3

A FEW ILLUSTRATIVE EXPERIMENTS ON CASE 1 SHOWING THE EFFECT OF POSTURE UPON THE MIXING WITH BLOOD OF INTRAVENOUSLY INJECTED BRILLIANT VITAL RED

DATE	TIME OF ASSUMING POSITION	TIME OF DYE INJECTION	TIME OF OBSERVATION	CONCENTRATION OF DYE* IN PLASMA IN THE VENOUS BLOOD OF RIGHT ARM %	LEFT ARM %	FOOT %
			Lying Down			
5/ 7/27	9.00 A.M.	1.30 P.M.	1.35 P.M.	99.2		98.9
5/17/27	10.30 A.M.	1.00 P.M.	1.06 P.M.	103.0	101.9	102.1
6/ 9/27	11.00 A.M.	2.06 P.M.	2.11½ P.M.	98.9	98.0	97.5
			2.19 P.M.	98.4	97.7	96.9
			Standing Still			
6/ 7/27	1.09 P.M.	1.17 P.M.	1.23 P.M.	123.0	115.8	101.1
			1.31½ P.M.	121.0	117.1	119.6
			1.48 P.M.	116.9	115.0	120.2
6/16/27	2.58 P.M.	3.12½ P.M.	3.19½ P.M.	120.0	118.4	77.5
			3.27 P.M.	110.8	109.0	103.6
			3.56 P.M.	119.0	109.5	117.6
7/14/27	2.08½ P.M.	2.12½ P.M.	2.18 P. M.	125.0	124.4	67.0
			2.24 P.M.	118.7	114.1	111.8
			2.30 P.M.	116.6	113.0	120.2

* See footnote to table 2.

TABLE 4

THE EFFECT OF POSTURE UPON THE VOLUME OF CELLS IN BLOOD SIMULTANEOUSLY WITHDRAWN FROM RIGHT ARM, LEFT ARM AND FOOT VEINS

CASE NO.	AVERAGE NO. OF CC. OF CELLS PER 100 CC. OF BLOOD: LYING DOWN RIGHT ARM VENOUS BLOOD	LEFT ARM VENOUS BLOOD	FOOT VENOUS VLOOD	STANDING STILL RIGHT ARM VENOUS BLOOD	LEFT ARM VENOUS BLOOD	FOOT VENOUS BLOOD
1	44.6	44.7	44.5	49.0	49.1	50.6
3	39.4	40.8	40.6	43.5	43.8	45.5
5	49.9	48.1	49.1	51.0	50.5	51.7
Average	44.6	44.5	44.7	47.8	47.8	49.3

TABLE 5

CASE NO.	AVERAGE RED CELL COUNT FOR STANDING STILL POSITION: OBSERVED MLNS. PER CU. MM.	CALCULATED FROM CHANGE IN TOTAL BLD. VOL. MLNS. PER CU. MM.	RATIO OBS./CALC.	AVERAGE PLASMA PROTEIN FOR STANDING STILL POSITION: OBSERVED %	CALCULATED FROM PLASMA VOLUME CHANGE %	RATIO OBS./CALC.	AVERAGE PLASMA WATER FOR STANDING STILL POSITION: OBSERVED %	CALCULATED FROM PLASMA VOLUME CHANGE %	RATIO OBS./CALC.
1	5.7	5.2	1.096						
2	4.4	4.7	0.937						
3	4.6	4.3	1.070	8.3	8.1	1.023	93.0	92.7	1.002
4A	4.3	4.3	1.000	8.2	7.9	1.038	93.0	93.0	1.000
5	5.1	5.1	1.000	7.8	8.1	0.963	93.4	93.3	1.001
6	4.6	4.4	1.045	7.9	7.2	1.098	93.0	93.2	0.998
Average	4.8	4.7	1.023	8.1	7.8	1.030	93.1	93.1	1.000

TABLE 6

CASE NO.	CC. OF CELLS PER 100 CC. OF BLOOD: LYING	STANDING	TOTAL PLASMA VOLUME (CC.) (BY DYE METHOD): LYING	STANDING	DIFF.	CALCULATED TOTAL BLOOD VOLUME (CC.): LYING	STANDING	DIFF.	TOTAL CELL VOLUME (CC.): LYING	STANDING	DIFF.
1	44.6	49.6	3055	2580	−475	5520	5120	−400	2465	2540	+75
2	45.9	48.8	2770	2445	−325	5120	4780	−340	2350	2335	−15
3	40.1	44.2	2140	1900	−240	3575	3405	−170	1430	1505	+75
4A	35.6	38.0	2495	2195	−300	3880	3540	−340	1385	1345	−40
5	47.8	50.9	2505	2160	−345	4800	4400	−400	2295	2240	−55
6	28.1	30.2	2755	2485	−270	3835	3560	−275	1080	1075	− 5
Average					−325			−320			+ 5

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[2] Research Fellow in Medicine, Massachusetts General Hospital.

[3] The time varied from 25 to 60 minutes.

STERILITY IN AMERICAN MARRIAGES

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Attempts have been made to estimate the proportion of marriages that remain without issue in certain sections of the American population. A summary of the principal inquiries on this subject, and of their results, has been given by Reynolds and Macomber in their book "Fertility and Sterility in Human Marriage"(**1924**). The results cannot be said to be very satisfactory, for the figures given range from eight to eighteen per cent. In many instances they are based on relatively small and perhap not very representative samples of population. In one case (American army draft) all marriages of a duration of two years or more were included in the sample, and so a marriage of only two years standing was recorded as childless if there was no issue within that period. This limit of two years is altogether too low. For example, in Australia[1] (no American statistics are available) it has been found that as many as twenty per cent of all births come after a lapse of two years or more. Thus many of the marriages, which in the records of the army draft were put down as childless, would in subsequent years have had issue.

It will now be shown how, by a suitable method, information regarding the childlessness in American marriages can be extracted indirectly from the data published by the United States Census Bureau. The results so obtained have the two-fold advantage of being based on a large sample (all the births of one calendar year in the Birth Registration Area, except the State of Maine), and of being quite independent of any arbitrary limit of the duration of marriage, in determining the cases to be included in the estimate.

We proceed as follows:

1. *Proportion of Females Born That Eventually Marry.*—In table 1 the first column sets forth successive ages of life; column 2 gives, for each age or age group, the proportion of white females single (never married), according to United States Census for the year 1920. Column 3 shows the deaths in each year (or group of years) in a cohort of 100,000 white females, counted at birth, according to the life table for 1919–1920 (E. Foudray) published by the Census Bureau, and interpolated for individual years by King's method.[2] Column 4, which contains the cross products of columns 2 and 3, gives the white females that die single in each year (or year-group) of life, out of the cohort of 100,000 born. The total of column 4 shows that out of 100,000 born, 21,793 die single and 78,207 marry eventually. This is, of course, on the assumption that the death-rate among the married

and the single is the same—an assumption which is admittedly not quite true to facts; but the error involved is probably not great.

2. *Proportion of Females Born That Eventually Become Legitimate Mothers.*—To determine the number of females that eventually become legitimate mothers we make use of the fact that every wife who has a first-born legitimate child is a legitimate mother and, conversely, every legitimate mother has a first-born legitimate child.

The number of first children born to white women, classified by age groups, for the year 1920, is given in the Sixth Annual Report of Birth Statistics (1920), issued by the Census Bureau, on page 201 (table 7). The figures there given require correction, however, for our present purpose, on account of illegitimate births. We may, without serious error, put down all illegitimate births as first births. For, in the first place a large proportion of illegitimate births are actually first births. Secondly, of those illegitimate births that may not be first births, those by married women will in most cases be reported as legitimate; on the other hand, illegitimate births out of wedlock will, in the majorty of cases, be reported as *first* births even if not such in fact. Hence we shall, without serious error, deduct *all* illegitimate births from the total first births to obtain the legitimate first births.[3]

The number of first-born legitimate children per annum per 100,000 women of stated age (in quinquennial age groups) thus obtained is set forth in column 2 of table 3. Column 3 shows the value of the survival factor (the l_x of actuarial notation), according to the Foudray life table, for the central epoch of each quinquennial age group. The sum of column 4, which is the sum of the cross products of columns 2 and 3, when multiplied by five, gives the number of legitimate first children born of the surviving members of the cohort of 100,000 white females, as they go through life. Obviously, this same total, namely, 64,830, also gives the number of white females out of the initial cohort of 100,000 that eventually have a first child, i.e., that eventually become mothers.

It is thus found from table 1 that out of 100,000 white females born, 78,207 marry and from table 3, 64,830 become mothers. That is to say, 82.9 per cent of women that marry, eventually become mothers, and 17.1 per cent die childless. We may, therefore, say that the *gross* or *effective sterility* (childlessness) of American women according to the vital statistics of 1920 was 17.1 per cent; that is to say, 17.1 per cent of all women that marry remain without issue for one reason or another.

3. *Net Sterility.*—The figure for the *gross* sterility obtained as set forth above contains a component foreign to sterility as commonly understood. Some marriages are without issue, not from any inherent sterility of husband or wife, but simply because wife or husband or both die before there has been reasonable time for issue to be born. It is desirable to ascertain

at least the order of magnitude of this component. We shall proceed on the assumption that the principle of superposition of small disturbances is here applicable. We shall first obtain an estimate of the effect of the premature death of wives, second, an estimate of the effect of the premature death of husbands, and by adding these two we shall obtain our estimate of the total effect of such premature deaths in the number of marriages without issue. Similarly the effect of divorces will be estimated.

4. *Effect of Premature Death of Wives.*—Column 5 of table 1 shows the survivors that would result at each age of life, out of an initial cohort of 100,000 (white) females born, if the *single* persons died according to the actual 1919–1920 life table, *but the married females did not die until they reached their 55th year.* Column 8 of the same table shows the number of first-born children, and hence of mothers, that would result in these circumstances. It is seen that this number would be 65,800. The proportion of married women that would become mothers would, then, be 84.1 per cent and the childlessness would be 15.9 per cent, as compared with 17.1 per cent when deaths take place normally according to the life table. Thus 1.2 per cent of marriages remain childless through premature death of wife, whereas they would have led to issue had these wives lived to age 55.

5. *Effect of Premature Death of Husbands.*—The computation of the effect of premature deaths of husbands upon the proportion of married women that eventually become mothers offers special difficulties. We have no statistics of children born, classified by ordinal number of child and *age of father.* The Report on Statistics of Births for 1920 does, however, give in table 6, page 193, the number of births from couples classified by age of husband and age of wife. If we assume that the proportion of children born at the several ages of the parents, as shown for *all* children, holds approximately also for *first children*, we can construct a table giving number of the wives' first-born children, classified by age of fathers. For our purposes the error contained in the assumption will not be very serious since we are concerned only with a correcting term, and errors therein are of the nature of second order errors, as related to the sterility itself.

Table 2 is similar in general character to table 1, but relates to a cohort of 100,000 white males instead of females. Column 5 here also shows the number of survivors (husbands in this case) that would be left over at successive ages of life if no deaths occurred among the married. Column 8 gives the number of first-born, and, therefore, the number of mothers, that would result if we attribute one wife to each husband. On the same supposition the number of wives would be the same as the number of husbands, namely, 74,099. We thus find for the ratio $\frac{\text{mothers}}{\text{wives}}$ the figure $\frac{62,860}{74,099} = 0.848$, or 15.2 per cent for the sterility. Comparing this figure

TABLE 1

NUMBER OF WHITE FEMALES DYING SINGLE IN A LIFE TABLE COHORT OF 100,000, TOGETHER WITH NUMBER OF LEGITIMATE FIRST-BORN CHILDREN THAT WOULD RESULT IF NO DEATHS OCCURRED AMONG THE MARRIED UNTIL THE END OF THE REPRODUCTIVE PERIOD (TO AGE 55)

(1) AGE PERIOD	(2) PER CENT SINGLE[1]	(3) LIFE TABLE DEATHS[2]	(4) DEATHS SINGLE COLS. 2 × 3	(5) SURVIVORS TO END OF AGE PERIOD PLUS DEATHS MARRIED	(6) SURVIVORS PLUS DEATHS MARRIED, AT PIVOTAL AGE	(7) LEGIT. 1ST BIRTHS PER 100,000 WHITE FEMALES[3] PER ANNUM	(8) NO. OF 1ST BIRTHS = NO. OF MOTHERS IN COHORT COLS. 6 × 7
0–15	100.0	11,904	11,904	88,096		9	8
15–16	98.6	236	233	87,863			
16–17	95.6	271	259	87,604			
17–18	88.1	302	266	87,338	87,204	3,358	2,928
18–19	80.5	334	269	87,069			
19–20	71.2	370	263	86,806			
20–21	61.9	404	250	86,556			
21–22	54.0	438	237	86,319			
22–23	47.5	465	221	86,098	85,999	6,836	5,879
23–24	40.7	486	198	85,900			
24–25	34.9	506	177	85,723			
25–26	30.8	520	160	85,563			
26–27	27.2	533	145	85,418			
27–28	24.0	543	130	85,288	85,230	3,421	2,916
28–29	21.2	552	117	85,171			
29–30	19.1	556	106	85,065			
30–31	17.9	559	100	84,965			
31–32	16.3	560	91	84,874			
32–33	15.4	561	86	84,788	84,747	1,212	1,027
33–34	14.6	559	82	84,706			
34–35	14.0	555	78	84,628			
35–36	13.7	550	75	84,553			
36–37	13.2	547	72	84,481			
37–38	12.8	545	70	84,411	84,377	389	328
38–39	12.5	546	68	84,343			

39–40	12.2	547	67	84,276			
40–41	12.0	550	66	84,210			
41–42	11.8	554	65	84,145			
42–43	11.6	566	66	84,079	84,046	83	70
43–44	11.4	580	66	84,013			
44–45	11.2	598	67	83,946			
45–46	11.0	618	68	83,878			
46–47	10.8	641	69	83,809			
47–48	10.6	669	71	83,738	83,702	4	3
48–49	10.4	698	73	83,665			
49–50	10.2	729	74	83,591			
50–51	10.0	764	76	83,515			
51–52	9.9	801	79	83,436			
52–53	9.8	847	83	83,353	83,310	1	1
53–54	9.7	897	87	83,366			
54–55	9.5	952	90	83,176			
0–55			16,824				13,160
55–65	8.7	12,937	1,126				×5
65–	7.3	52,650	3,843				
Deaths single, all ages			21,793	Number of legitimate first-born children or number of mothers in cohort			65,800
Deaths married[4]			78,207				

$$\frac{\text{Mothers in cohort}}{\text{Deaths married}} = \frac{65{,}800}{78{,}207} = 0.841$$

$$\text{Childlessness} = 1 - 0.841 = 0.159 = 15.9 \text{ per cent}$$

[1] Interpolated for individual years of age from Report of U. S. Census, 1920, Population, Vol. II, p. 390, table 3.

[2] E. Foudray's Abridged U. S. Life Table for 1919–20, interpolated for individual years by King's method. (Supplement to Seventy-Fifth Annual Report of Registrar General of Births, etc., for England and Wales, 1914.)

[3] The figures in this column refer to a basis of 100,000 white females *of all marital states* and of the age period indicated in column 1.

[4] "Married" here means "Ever Married," and includes widowed and divorced.

TABLE 2

NUMBER OF WHITE MALES DYING SINGLE IN A LIFE TABLE COHORT OF 100,000 TOGETHER WITH THE NUMBER OF THEIR WIVES'[1] LEGITIMATE FIRST-BORN CHILDREN THAT WOULD RESULT IF NO DEATHS OCCURRED WITHIN THE REPRODUCTIVE PERIOD OF THE HUSBANDS (TO AGE 65)

(1) AGE PERIOD	(2) PER CENT SINGLE[2]	(3) LIFE TABLE DEATHS[3]	(4) DEATHS SINGLE IN LIFE TABLE COLS. 2 × 3	(5) SURVIVORS PLUS DEATHS MARRIED TO END OF AGE PERIOD	(6) SURVIVORS PLUS DEATHS MARRIED AT PIVOTAL AGE	(7) LEGITIMATE 1ST BIRTHS PER 100,000 FEMALES[4] CORRESPONDING TO HUSBANDS IN THE STATED AGE GROUPS	(8) NO. OF 1ST BIRTHS = NO. OF MOTHERS CORRESPONDING TO COHORT COLS. 6 × 7
0–15	100.0	14,143	14,143	85,857			
15–16	99.8	271	270	85,587			
16–17	99.7	306	305	85,282			
17–18	98.0	335	328	84,954	84,785	377	320
18–19	95.2	355	338	84,616			
19–20	90.9	372	338	84,278			
20–21	85.4	386	330	83,948			
21–22	78.7	398	313	83,635			
22–23	72.6	413	300	83,335	83,195	4,418	3,676
23–24	66.0	425	281	83,054			
24–25	58.8	436	256	82,798			
25–26	51.9	447	232	82,566			
26–27	45.7	458	209	82,357			
27–28	40.4	470	190	82,167	82,081	5,237	4,299
28–29	35.7	485	173	81,994			
29–30	31.9	499	159	81,835			
30–31	28.9	515	149	81,686			
31–32	26.4	529	140	81,546			
32–33	24.4	540	132	81,414	81,352	2,930	2,384
33–34	22.7	549	125	81,289			
34–35	21.3	556	118	81,171			
35–36	20.1	562	113	81,058			
36–37	19.0	568	108	80,950			
37–38	18.2	576	105	80,845	80,795	1,464	1,183
38–39	17.3	585	101	80,744			
39–40	16.7	595	99	80,645			
40–41	16.1	606	98	80,547			
41–42	15.5	617	96	80,451			
42–43	15.1	631	95	80,356	80,309	547	439
43–44	14.7	646	95	80,261			
44–45	14.3	661	95	80,166			
45–46	13.8	677	93	80,073			
46–47	13.6	698	95	79,978	79,835	228	182
47–48	13.2	723	95	79,883			
48–49	12.9	752	97	79,786			

42–43	15.1	631	95	80,451			
43–44	14.7	646	95	80,356	80,309	547	439
44–45	14.3	661	95	80,261			
45–46	13.8	677	93	80,166			
46–47	13.6	698	95	80,073			
47–48	13.2	723	95	79,978	79,835	228	182
48–49	12.9	752	97	79,883			
49–50	12.6	783	99	79,786			
50–51	12.2	820	100	79,687			
51–52	11.9	861	102	79,587			
52–53	11.6	910	106	79,485	79,325	73	58
53–54	11.3	966	109	79,379			
54–55	11.1	1,028	114	79,270			
55–56	10.8	1,094	118	79,156			
56–57	10.6	1,163	123	79,038			
57–58	10.4	1,227	128	78,915	78,722	30	24
58–59	10.2	1,286	131	78,787			
59–60	10.1	1,340	135	78,656			
60–61	9.9	1,395	138	78,521			
61–62	9.8	1,450	142	78,383			
62–63	9.7	1,528	148	78,241	78,016	9	7
63–64	9.6	1,609	154	78,093			
64–65	9.5	1,696	161	77,939			
0–65			22,222	77,778			12,572
65–	7.5	49,059	3,679				×5
Deaths single, all ages			25,901	Number of legitimate first-born children, or number of mothers, corresponding to husbands in cohort			62,860
Deaths married			74,099				

$$\frac{\text{Mothers corresponding to husbands}}{\text{Deaths married}} = \frac{62,860}{74,099} = 0.848$$

Childlessness = 1 − 0.848 = 0.152 = 15.2 per cent (corrected for deaths of husbands within reproductive period of their wives).

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[1] The wife's first-born child may not be the husband's first-born under actually existing conditions, since he may be a remarried widower. In the hypothetical case of "no deaths among the married" this distinction would disappear (except in the case of divorced persons) since there would be no widows and widowers. The number of husbands would then be equal to the number of wives.

[2] Interpolated for individual years of age from Report of U. S. Census, 1920, Population, Vol. II, p. 390, table 3.

[3] E. Foudray's Abridged U. S. Life Table for 1919–20, interpolated for individual years by King's method. (Supplement to Seventy-Fifth Annual Report of Registrar General of Births, etc., for England and Wales, 1914.)

[4] The figures in this column refer to a basis of 100,000 white females *of all marital states*. The correspondence referred to is the correspondence in age of husband and wife, as exhibited in table 4.

TABLE 3

AGE SCHEDULE OF LEGITIMATE FIRST-CHILD-MATERNITY FREQUENCY AMONG WHITE FEMALES AND CORRESPONDING NUMBER OF FIRST-BIRTHS RESULTING FROM A LIFE TABLE COHORT OF 100,000 WHITE FEMALES; ALSO CORRESPONDING FREQUENCIES WITH WIDOWED AND DIVORCED RESTORED TO MARITAL STATE

(1) AGE GROUP	(2) LEGITIMATE 1ST BIRTHS PER 100,000 WHITE FEMALES PER ANNUM	(3) SURVIVORS TO MIDPOINT OF AGE GROUP	(4) RESULTING LEGIT. FIRST-BIRTHS COLS. 2 × 3	(5) PER CENT[1] MARRIED	(6) PER CENT[1] WIDOWED	(7) PER CENT[1] DIVORCED	(8) LEGITIMATE 1ST BIRTHS WITH WIDOWS RESTORED COL. $\frac{5+6}{5} \times 4$	(9) LEGITIMATE 1ST BIRTHS WITH DIVORCED RESTORED COL. $\frac{5+7}{5} \times 4$
10–15	9	88,567	8	...	...	...	8	8
15–20	3,358	87,438	2,936	11.5	0.2	0.1	2,987	2,962
20–25	6,836	85,509	5,845	50.8	1.0	0.5	5,961	5,903
25–30	3,421	82,960	2,838	73.1	2.1	0.8	2,919	2,869
30–35	1,212	80,181	972	80.3	3.3	0.9	1,012	983
35–40	389	77,417	301	81.4	5.2	1.0	321	305
40–45	83	74,664	62	80.0	7.8	1.0	68	63
45–50	4	71,610	3	76.8	11.9	1.0	4	3
50–55	1	67,860	1	72.0	17.8	0.9	1	1
			12,966				13,281	13,097
			× 5				× 5	× 5
Mothers in cohort			64,830				66,405	65,485
Wives in cohort (see Table 1)			78,207					

Gross sterility (childlessness) of wives $1 - \frac{64,830}{78,207} = 17.1$ per cent

Childlessness of wives due to causes other than death of husband $1 - \frac{66,405}{78,207} = 15.1$ per cent

Childlessness of wives due to causes other than divorce $1 - \frac{65,485}{78,207} = 16.3$ per cent

Childlessness of wives due to premature death of husband, according to method of table 3 $= 17.1 - 15.1 = 2.0$ per cent

Childlessness of wives due to premature death of husband, according to method of table 2 $= 17.1 - 15.2 = 1.9$ per cent

Childlessness of wives due to divorce	= 17.1 − 16.3 = 0.8 per cent
Childlessness of wives due to premature death of wives (see Table 1)	= 17.1 − 15.9 = 1.2 per cent
Net Sterility	= 17.1 − 2.0 − 0.8 − 1.2 = 13.1 per cent

[1] Report of United States Census, 1920, Population, Vol. II, p. 390, table 3.

TABLE 4

DISTRIBUTION OF LEGITIMATE FIRST-BIRTHS OF WIVES, BY AGE OF HUSBAND

AGE OF MOTHER	LEGITIMATE FIRST-BIRTHS PER 100,000 WHITE FEMALES[1] PER ANNUM	AGE OF FATHER 10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55 and over
10–55	15,313	..	377	4,418	5,237	2,930	1,464	547	228	73	39
10–15	9	..	3	4	1	1	...	...	..	..	.
15–20	3,358	..	316	1,920	818	203	68	18	9	3	3
20–25	6,836	..	56	2,325	2,964	1,035	316	85	35	12	8
25–30	3,421	..	2	159	1,356	1,200	504	128	48	15	9
30–35	1,212	..	...	9	91	457	420	159	52	16	8
35–40	389	..	...	1	7	32	148	124	55	15	7
40–45	83	..	...	...	...	2	8	32	27	10	4
45–50	4	..	...	...	...	...	...	1	2	1	.
50–55	1	..	...	...	...	...	...	...	..	1	.

[1] These figures relate to white females of all marital states combined.

with the 17.1 per cent gross or effective sterility when premature deaths are not prevented, shows a balance of 1.9 per cent of childless marriages due to the premature death of husbands.

The assumption of one wife per husband might not be strictly in accordance with facts, owing to the remarriage of widowed persons. This assumption is not, however, really necessary for our argument. If there were n wives per husband, the figures in column 8 would all have to be multiplied by n; but at the same time the total number of wives would then be 74,099 n instead of 74,099; and hence in the ratio of mothers to wives, the factor n would cancel out.

In point of fact, however, the assumption $n = 1$ represents very nearly indeed the actual situation as defined by the Census statistics here employed. For we have found that a cohort of 100,000 females resulted in 78,207 wives, and a cohort of 100,000 males resulted in 74,099 husbands. If the actual number of husbands is to be equal to the number of wives, a cohort of $100{,}000\,\frac{78{,}207}{74{,}099}$ males must be associated with a cohort of 100,000 females. This gives for the ratio of male to female births the figure 1.055. The figure actually observed in 1920, in the white population was 1.059; a near approach to the figure 1.055 which would make $n = 1$.

A possible objection to the argument presented above should be met here. It might be objected that restoring the dying husbands and wives would disturb the existing marital ratios. But if *both* sexes were so restored (as we have here supposed in applying the principle of superposition of two disturbances) the marital ratios would not be very greatly changed.

5. *Effect of Premature Death of Husbands. Alternative Method.*—The following method of obtaining an estimate of the effect of premature deaths of husbands is perhaps somewhat more direct than that described in section 4; it also serves to give an independent confirmation of the results obtained.

Column 8 of table 3 is essentially similar in character to column 4 of the same table, except that there have been added, in each age group, to the first-born children resulting from the married women a proportionate number (see columns 8 and 9) of first-born children, such as would result from the widowed women if they were still married. So, for example, in the age group 20–25, the per cent of married white females is 50.8, the per cent of widowed white females is 1.0, and the number of resulting first-born children from the married white females in this age group is 5845 per annum. With the widows restored to the married state, the number of first-born in this age group of mothers would be $5845 \times \frac{(50.8 + 1.0)}{50.8}$ = 5961. Making similar adjustments in the remaining age groups, and

forming the total as before, we find that out of a cohort of 100,000 white females born, if none lost their husbands by death, 66,405 would eventually become mothers. In these circumstances the resultant gross sterility is found, by a similar computation as before, to be 15.1. In other words, 2.0 per cent of the women that marry remain without issue merely owing to the premature death of their husbands; these women would eventually have had a child if their husbands had lived to the end of the reproductive period of the wife.

This result is in close agreement with that obtained by the first method, namely, 1.9 per cent.

6. *Effect of Divorces.*—By a method precisely analogous to that described in section 5 the contribution of divorces to the *effective* or *gross* sterility is found to be 0.8 per cent.

7. *Net Sterility.*—Collecting our results, we find the net sterility by deducting from the gross 17.1 per cent, a portion 1.2 referable to premature deaths of wives, 2.0 referable to premature deaths of husbands, and 0.8 referable to divorces, giving a net sterility of 13.1.

It is interesting to compare this figure of 13.1 per cent with the findings of the Immigration Commission, as cited by Ellsworth Huntington, "Builders of America," 1927, p. 30, and originally published in Volume 28 of Senate Document No. 282, pp. 744, 762 and 783, tables 5, 15 and 26. These show for certain cities and rural areas of the States of Rhode Island, Ohio and Minnesota, 13.14 per cent of childless marriages of 10 to 19 years' duration, in a sample of 15,953 native white women of native parentage, in the year 1900.

Summary.—The *effective* or *gross* sterility of marriages[5] of American (white) wives has been computed as 17.1 per cent. Of this about 1.2 per cent is due to the premature death of wives who otherwise would eventually have had a child; 2.0 per cent further is due to premature deaths of husbands whose wives would otherwise eventually have had a child. A further 0.8 per cent is due to childlessness resulting from divorces in couples that would eventually have had a child.

The *net* sterility of marriages of American (white) wives is accordingly $17.1-1.2-2.0-0.8 = 13.1$ per cent.

[1] *Official Yearbook of the Commonwealth of Australia*, **1925**, page 972.

[2] Supplement of the Seventy-Fifth Annual Report of the Registrar General of Births, Marriages and Deaths for England and Wales, 1914.

[3] This same assumption has also been made by C. H. Wickens in the *Official Yearbook of Australia*, **1925**, p. 972.

[4] *Birth, Stillbirth and Infant Mortality Statistics for the Birth Registration Area of the U. S.*, **1924** (published by the Census Bureau), p. 9.

[5] In this estimate no attempt is made to determine which of the two partners in marriage is the cause of the sterility.

THE PHOTOCHEMICAL UNION OF HYDROGEN AND CHLORINE

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The union of hydrogen and chlorine under the influence of blue light, with an excess of hydrogen, and in the presence of small amounts of oxygen and water vapor, has been found to be expressed by the formula[1]

$$\frac{d(\mathrm{HCl})}{dt} = k\,\frac{(\mathrm{Cl_2})^2}{(\mathrm{O_2})}\,.$$

Under such conditions, Kornfeld and Müller[2] were able to obtain a yield of about 10^4 molecules of hydrogen chloride per quantum of light. Bodenstein[1] had already estimated a yield of 10^6 molecules per quantum of light. The object of the present investigation was to determine the quantum yield and to obtain more facts concerning the kinetics of the reaction, using blue light, and under conditions affording a *minimum concentration of oxygen.*

The reaction was carried out in an apparatus entirely of quartz, and conditions ensuring the absence of impurities, particularly stopcock grease, mercury vapor, etc. The usual stopcocks were replaced by solid chlorine capillary stopcocks described by Thon.[3] However, it was necessary to modify the procedure recommended by Thon, because, whereas he was able to use pure chlorine in his capillary stopcocks, the present investigation necessitated the use of a mixture of hydrogen and chlorine in the capillary stopcocks.

Chlorine was freed of oxygen, by washing a sample of fractionally distilled liquid chlorine, maintained at −78°C., with electrolytic, purified hydrogen, continuously for a period of six weeks. This chlorine was stored in a quartz gas-wash flask, connected to the rest of the quartz apparatus, and was used for all the experiments. The hydrogen-chlorine mixture, leaving the wash flask, passed in turn through a capillary stopcock, reaction vessel, capillary stopcock, liquid air trap, and, then, either to a Bunsen absorber or the vacuum line. The ratio of chlorine to hydrogen, for each investigation, was varied by changing the temperature of the alcohol–carbon-dioxide bath containing the wash flask. Pressures were measured with a Bodenstein quartz gage. The hydrogen pressure was determined by freezing out the chlorine and hydrogen chloride in the reaction chamber with liquid air. The temperature of the liquid air was measured with an oxygen thermometer.

The reaction vessel was illuminated with light of wave-length greater than 4050 Å. Light from a 500-watt "Nitra" lamp, maintained at a

constant voltage, passed through a filter of ferrous sulphate, and one of copper ammonium sulphate, a lead glass plate and, finally, a ground glass plate, which served to illuminate the reaction vessel uniformly.

The deflection of a very sensitive galvanometer, produced by the illumination of a thermopile, placed directly behind the reaction vessel, served as a measure of the energy. This system permitted the measurement of 0,432 ergs per square centimeter per second. However, the amount of energy absorbed by the chlorine (even with a pressure greater than one atmosphere) was so small that no difference in the deflection could be detected with the reaction vessel filled with chlorine or evacuated. It is only possible, therefore, to calculate a *lower limit* of the quantum yield. Since the area of illuminated surface was 110 cm.^{2}, the *maximum* energy absorbed would be 47,5 ergs per second. Assuming a mean effective wave-length of 4360 Å, 1 erg = $2{,}2 \times 10^{11}$ $h\nu$, and 47,5 ergs = $104{,}5 \times 10^{11}$ $h\nu$. The most sensitive mixture (Cl_2p = 334 mm.; H_2p = 411 mm. at 0°C.) investigated, yielded 35 mm. of hydrogen chloride per minute in a vessel of 300 cc., or $6{,}27 \times 10^{18}$ molecules of hydrogen chloride per second.

This gives a *minimum yield of* 6×10^5 *molecules of hydrogen chloride per quantum of light.* The actual yield obtained, here, may well be ten times this value.

Although this value is greater than heretofore reported, it is by no means to be considered as an upper limit of the quantum yield for this reaction. There action velocity constant, k', using the formula

$$\frac{\frac{d(HCl)}{dt}}{(Cl_2)^2} = k', \text{ remained constant for each run.}$$

However, the value of k' for runs at different initial pressures of chlorine showed a definite trend—the higher the initial value of the chlorine pressure, the lower the value of k'. For example, the value of k' for an initial pressure of 150 mms. of chlorine was about 2,5 times as great as the k' for an initial pressure of 335 mms. of chlorine. This was, no doubt, due to the presence of some oxygen in the gas mixture—very likely of the order of 1/1000 of a per cent. Under the present experimental conditions, the oxygen pressure would increase with increasing chlorine pressure and the data obtained would seem to be in accordance with the Bodenstein-Dux equation,[1] above. Since the ratio of hydrogen to chlorine was always greater than unity, the formulae proposed by Cremer[4] for the hydrogen-chlorine reaction, cannot be applied, easily, to these measurements. Further work will, undoubtedly, lower the oxygen concentration and increase the quantum yield.

Summarizing, a study was made of the photochemical union of hydrogen and chlorine under conditions affording a minimum concentration of

oxygen. A yield greater than 6×10^5 molecules of hydrogen chloride per quanta of light was obtained. The reaction velocity measurements seem to be in accord with the Bodenstein-Dux equation:

$$\frac{d(HCl)}{dt} = \frac{(Cl_2)^2}{(O_2)}.$$

This work was performed in the physikalisch-chemischen Institut der Universität Berlin, under the direction of Professor Max Bodenstein. The work was much facilitated due to the efforts of my predecessor, Dr. P. B. Ganguli, who assembled much of the apparatus and carried out some preliminary measurements.

I wish to take this opportunity of thanking Professor Bodenstein for his kindness in placing the facilities of his laboratory at my disposal and for his ever helpful assistance.

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[1] Bodenstein u. Dux, *Zeitschr. physik. Chem.*, **85**, 297 (1913).
[2] Kornfeld u. Müller, *Ibid.*, **117**, 242 (1925).
[3] W. Thon, *Ibid.*, **124**, 327 (1926).
[4] E. Cremer, *Ibid.*, **128**, 285 (1927).